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Steady Quadratic Stokes Flow Past Deformed Sphere: A Novel Perturbation Technique

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Abstract

In this paper, the problem of steady quadratic Stokes flow past a deformed sphere has been tackled with the use of a novel perturbation technique in both situations when a uniform stream is along the axis of symmetry (axial flow) and when it is perpendicular to the axis of symmetry (transverse flow). The most general form of the deformed sphere, governed by polar equation, is considered here for the study. The general expressions for axial and transverse Stokes drag for deformed sphere has been derived up to the second order of deformation parameter for parabolic and stagnation like parabolic flow. The class of prolate, oblate and egg-shaped axisymmetric bodies is considered for the validation and further numerical discussions. The numerical values of the drag coefficients and their ratio have been evaluated for the various values of deformation parameters, eccentricity and aspect ratio and corresponding variations are depicted via graphs.

Keywords: Quadratic Stokes flow; deformed sphere; prolate; oblate and egg-shaped axially symmetric bodies

MSC 2010 No.: 75D05, 76D07

1. Introduction

In physical and biological science, and in engineering, there is a wide range of problems of interest like sedimentation problem, lubrication processes concerning the flow of a viscous fluid in which a solitary or a large number of bodies of microscopic scale are moving, either being carried about passively by the flow, such as solid particles in sedimentation, or moving actively as in the locomotion of micro-organisms. In the case of suspensions containing small particles, the presence of the particles will influence the bulk properties of the suspension, which is a subject of general interest in Rheology. In the motion of micro-organisms, the propulsion velocity depends critically on their body shapes and modes of motion, as evidenced in the flagellar and cilliary movements and their variations.

A common feature of these flow phenomena is that the motion of the small objects relative to the surrounding fluid has a small characteristic Reynolds number *Re*. Typical values of Re may range from order unity, for sand particles settling in water, for example, down to 10^{-2} to 10^{-6}, for various micro-organisms. In this low range of Reynolds numbers, the inertia of the surrounding fluid becomes insignificant compared with the viscous effects and is generally neglected and the Navier-Stokes equations of motion reduce to the Stokes equations as a first approximation. The zero Reynolds number flow is called Stokes flow. The hydromechanics of low Reynolds number flows play an important role in the study of rheology, lubrication theory, micro-organism locomotion and many areas of biophysical and geophysical interest. In the case when the inertial effects are negligible compared with the viscous forces, the Navier-stokes equations are usually simplified to Stokes equations as a first approximation. Determination of the solutions for the Stokes flows, however, is still recognized to be difficult in general for arbitrary body shapes. As a consequence, not many exact solutions are known. Of the few analytical methods available for solving Stokes flow problems, one is the boundary value method, which is based on the choice of an appropriate co-ordinate system to facilitate separation of the variables for the body geometry under consideration.

Another is the singularity method, whose accuracy depends largely on whether the correct types of singularity are used and how their spatial distributions are chosen. The boundary value method seems to have been widely adopted in practice, more so than the singularity method. Stokes flow of an arbitrary body is of interest in biological phenomena and chemical engineering. In fact, the body with a simple form such as a sphere or ellipsoid is less encountered in practice. The body, which is presented in science and technology, often takes a complex arbitrary form. For example, under normal condition, the erythrocyte is a biconcave disk in shape, which can easily change its form and present different contour in blood motion due to its deformability. In the second half of the twentieth century, a considerable progress has been made in treating the Stokes flow of an arbitrary body.

An exact solution for the motion of a spheroidal particle placed in a quadratic as well as in a linear flow of incompressible viscous fluid is very useful in the study of blood flow and general suspension rheology. In particular, a correct description of the behavior of a spheroidal particle in a paraboloidal or Poiseuille flow will facilitate accurate calculation of the bulk flow properties of tube flows of dilute or concentrated suspensions of blood cells, long-chain polymers or any other biological supra-macromolecules. When the Reynolds number based on the particle size, the local flow velocity and the kinematic viscosity of the surrounding fluid is very small, as in the case of microcirculation of blood cells, the inertial effects of the fluid can be neglected and the Navier-Stokes equations of motion reduce to the Stokes equations as a first approximation.

All these motions are characterized by low Reynolds numbers and are described by the solution of the Stokes equations. Although the Stokes equations are linear, to obtain their exact solutions for arbitrary body shapes or complicated flow conditions is still a formidable task. There are only relatively few problems in which it is possible to solve exactly the creeping motion equations for flow around a single isolated solid body. Stokes (1851) calculated the flow around a solid sphere undergoing uniform translation through a viscous fluid whilst Oberbeck (1876) solved the problem in which an ellipsoid translates through liquid at a constant speed in an arbitrary direction. Edwards (1892), applying the same technique, obtained the solution for the steady motion of a viscous fluid in which an ellipsoid is constrained to rotate about a principal axis. The motion of an ellipsoidal particle in a general linear flow of viscous fluid at low Reynolds number has been solved by Jeffery (1922), whose solution was also built up using ellipsoidal harmonics. The analysis described by Jeffery extended further by Taylor (1923). Goldstein (1929) obtained a force on a solid body moving through viscous fluid. Lighthill (1952) studied the problem of squirming motion of nearly spherical deformable bodies through liquids at very small Reynolds number. Hill and Power (1956) have obtained arbitrarily close approximations of the drag by proving a complimentary pair of extremum principles for a Newtonian viscous fluid in quasi-static flow.

In a series of studies over low-Reynolds-number-flow, Chwang and Wu (1974, part 1) have developed an effective method of solution for arbitrary body shapes. In this first part the authors have considered the viscous flow generated by pure rotation of an axisymmetric body having an arbitrary prolate form, the inertia forces being assumed to have a negligible effect on the flow. The method of solution explored in this paper is based on a spatial distribution of singular torques, called rotlets, by which the rotational motion of a given body can be represented. Exact expressions of torque are determined in closed form for a number of body shapes, including the dumbbell profile, elongated rods and some prolate forms. Chwang and Wu (1975, part 2) explored the fundamental singular solutions for Stokes flow that could be useful for constructing solutions over a wide range of free-stream profiles and body shapes.

We have employed these fundamental singularities (Stokeslet and their derivatives like rotlets, stresslets, potential doublets and higher order poles) to construct exact solutions to a number of exterior and interior Stokes-flow problems for several specific body shapes translating and rotating in a viscous fluid which may itself be providing a primary flow. The different primary flows considered here include the uniform stream, shear flows, parabolic profiles and extensional flows (hyperbolic profiles), while the body shapes cover prolate spheroids, spheres and circular cylinders. Chwang (1975, part 3) found exact solutions in closed form (expressions of drag) using the singularity method for various quadratic flows of an unbounded incompressible viscous fluid at low Reynolds numbers past a prolate spheroid with an arbitrary orientation with respect to the fluid. The quadratic flows considered here include unidirectional paraboloidal flows, with either an elliptic or a hyperbolic velocity distribution, and stagnation-like quadratic flows as typical representations. Chwang and Wu (1976, part 4) have analyzed the problem of a uniform transverse flow past a prolate spheroid of arbitrary aspect ratio at low Reynolds numbers by the method of matched asymptotic expansions. They have found expressions of drag depending on two Reynolds numbers, one based on the semi-minor axis b, $R_b=Ub/\nu$, and the other on the semi-major axis a, $R_a=Ua/\nu$. Johnson and Wu (1979, part 5) elucidated the characteristics of the general stokes flow for slender bodies of finite centre-line curvature. The singularity method for

Stokes flow has been employed by authors to construct the solutions to the flow past a slender torus. Ho and Leal (1976) considered the migration of a rigid sphere in a two-dimensional unidirectional shear flow of a second order fluid. Parker (1977) considered the two-dimensional motion of a projectile experiencing a constant gravitational force and a fluid drag force which is quadratic in the projectiles speed.

Majhi and Vasudevaiah (1982) considered the axisymmetric parabolic shear flow past a spinning sphere in an unbounded viscous medium using the matching technique and taking the non-uniform shear as the dominant feature, where the Reynolds number based on parabolic shear $R_e \leq 1$ and rotational Reynolds number based on the angular rotation of sphere $R_o \leq 1$ are such that $R_o^2/R_e = O(1)$. Kaloni (1983) studied the motion of a rigid sphere, suspended in micropolar fluid which is undergoing a slow unidirectional two-dimensional flow. Keh and Anderson (1984) obtained the configurational distribution function of dumbbell macromolecules (rigid and linearly elastic) in a quadratic rectilinear flow. Yuan and Wu (1987) obtained the analytic expressions in closed form for flow field by distributing continuously the image Sampsonlets with respect to the plane and by applying the constant density, linear and the parabolic approximation. Yang and Hong (1988) found exact solutions in closed form using the eigen function-expansion method for various linear and quadratic flows of an unbounded incompressible viscous liquid at low Reynolds number past a porous sphere with a uniform permeability distribution. Seki (1996) studied the motion of a rigid ellipsoidal particle freely suspended in a Poiseuille flow (parabolic flow) of an incompressible Newtonian fluid through a narrow tube numerically in the zero-Reynolds-number limit. Haber and Brenner (1984, 1999) investigated analytically the quasi-steady hydrodynamic Stokes drag force and torque exerted on each of N non-identical particles immersed in a general quadratic undisturbed flow field at infinity.

Datta and Srivastava (1999) advanced a new approach to evaluate the drag force in a simple way on a restricted axially symmetric body placed in a uniform stream (i) parallel to its axis, (ii) transverse to its axis, when the flow is governed by the Stokes equations under no-slip boundary conditions. Authors have evaluated the analytic closed form expressions of drag for spheroids, deformed sphere, cycloidal body, an egg-shaped body. Palaniappan and Daripa (2000) have found exact analytical solutions for the steady state creeping flow in and around a vapor-liquid compound droplet, consisting of two orthogonally intersecting spheres of arbitrary radii (a and b), submerged in axisymmetric extensional (hyperbolic) and paraboloidal flows of fluid with viscosity μ. Lin et al. (2005) studied the hydrodynamic interaction between two neutrally-buoyant smooth spheres moving at negligible Reynolds numbers in an unbounded plane Poiseuille flow calculated by three-dimensional boundary element (BEM) simulations. Pasol et al. (2006) provided comprehensive results for the creeping flow around a spherical particle in a viscous fluid close to a plane wall, when the external velocity is parallel to the wall and varies as a second degree polynomial in the coordinates. By using bipolar coordinates technique, authors have concluded that by linearity of Stokes equations, the solution is a sum of flows for typical unperturbed flows: a pure shear flow, a ‘modulated shear flow’, for which the rate of shear varies linearly in the direction normal to the wall, and a quadratic flow. Datta and Singhal (2011) tackled the problem of small Reynolds number steady flow past a sphere with a source at its centre. Prakash et al. (2012) studied the hydrodynamics of a porous sphere in an oscillatory viscous flow of an incompressible Newtonian fluid. Authors have derived the Faxen’s law for drag and torque acting on the surface of the porous sphere. In the later part of the paper, examples of uniform flow, oscillating Stokeslets, oscillatory shear flow and quadratic shear flow are discussed.

In the present paper, the author has tried to advance the self-developed conjecture [Datta and Srivastava (1999)] to evaluate the asymptotic expressions of drag over axi-symmetric bodies placed under different primary flows including uniform stream, parabolic profiles and stagnation like parabolic profiles by considering the surface average of the primary flow velocity explained by Chwang and Wu (1975, part 2 and 3). This conjecture is briefly explained in section 2. The reader is advisable to go through the author's (Srivastava et al., 2012) recently published paper for complete detail regarding its application to the class of oblate bodies placed in steady and uniform Stokes flow.

2. Body geometry and perturbation technique

Let us consider the axially symmetric body of characteristic length *L* placed along its axis (x-axis, say) in a uniform stream *U* of viscous fluid of density ρ_1 and kinematic viscosity ν. When Reynolds number *UL/ν* is small, the motion is governed by Stokes equations [Happel and Brenner (1964)],

$$\mathbf{0} = -\left(\frac{1}{\rho_1}\right) grad\ p + \nu\,\nabla^2\, \boldsymbol{u},\ div\, \boldsymbol{u} = 0, \tag{1}$$

subject to the no-slip boundary condition.

We have taken up the class of those axially symmetric bodies which possess continuously turning tangent, placed in a uniform stream *U* along the axis of symmetry (which is x-axis), as well as constant radius '*b*' of maximum circular cross-section at the mid of the body. This axi-symmetric body is obtained by the revolution of the meridional plane curve (depicted in Figure 1) about the axis of symmetry which obeys the following limitations:

i. Tangents at the points A, on the x-axis, must be vertical,
ii. Tangents at the points B, on the y-axis, must be horizontal, and
iii. The semi-transverse axis length 'b' must be fixed.

The point P on the curve may be represented by the Cartesian coordinates (x,y) or polar coordinates $(r,\ \theta)$ respectively , PN and PM are the length of the tangent and the normal at the point *P*. The symbol *R* stands for the intercepting length of normal between the point on the curve and point on axis of symmetry and α is the slope of the normal PM which can vary from 0 to π.

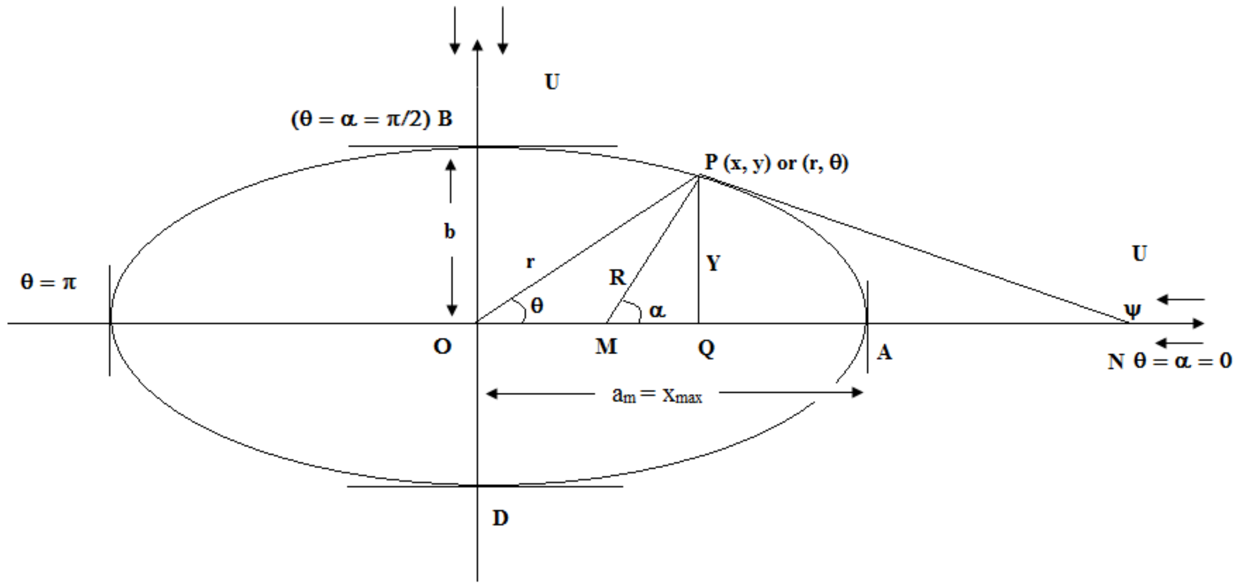

Figure 1 Geometry of axially symmetric body

2.1. Axial Flow

The expression of Stokes drag on such type of axially symmetric bodies placed in axial flow uniform flow parallel to the axis of symmetry) is given by [Datta and Srivastava (1999)]

$$F_{\|} = \frac{1}{2}\frac{\lambda b^2}{h_{\|}}, \quad \text{where } \lambda = 6\,\pi\,\mu\,U_{\| } \tag{2}$$

and

$$h_{\|} = \left(\frac{3}{8}\right)\int_0^{\pi} R\sin^3\,\alpha\,d\alpha, \tag{3}$$

where the suffix || has been introduced to assert that the force is in the axial direction.Sometimes it will be convenient to work in Cartesian co-ordinates. Therefore, referring to the Figure 1, for the profile geometry, we have

$$y\ =\ R\,\sin a, \quad tan\,a = -\left(\frac{dy}{dx}\right)^{-1} = -\frac{dx}{dy} = -x'. \tag{4}$$

Using above transformation, we may express (3) as

$$h_{\parallel} = -\frac{3}{4} \int_{0}^{a} \frac{yy''}{\left(1+y'^{2}\right)^{2}}\, dx, \tag{5}$$

where $2a_m$ represents the axial length of the body and dashes represents derivatives with respect to x. In the sequel, it will be found simpler to work with y as the independent variable. Thus, $h_{\parallel}$ assumes the form

$$h_{\parallel} = -\frac{3}{4} \int_{0}^{b} \frac{yx'^{2}x''}{\left(1+x'^{2}\right)^{2}}\, dy, \tag{6}$$

where dashes represents derivatives with respect to y.

2.2. Transverse flow

The expression of Stokes drag on such type of axially symmetric bodies placed in transverse flow (uniform flow perpendicular to the axis of symmetry) is given by [Datta and Srivastava (1999)]

$$F_{\perp} = \left(\frac{1}{2}\right)\frac{\lambda b^{2}}{h_{y}} \quad , \quad \text{where } \lambda = 6\pi\mu U_{\perp}, \tag{7}$$

and

$$h_{\perp} = \left(\frac{3}{16}\right)\int_{0}^{\pi} R\left(2\sin\alpha - \sin^{3}\alpha\right)\, d\alpha. \tag{8}$$

In the same manner as in the axial flow, equation (8) may also be written in Cartesian form as (in both cases having x and y treated as independent)

$$h_{\wedge} = -\frac{3}{8}\int_{0}^{a} \frac{yy''\left[1+2\left(y'\right)^{2}\right]}{\left[1+\left(y'\right)^{2}\right]^{2}}\, dx, \tag{9}$$

and

$$h_{\perp} = -\frac{3}{8}\int_{0}^{b} \frac{yx''\left[2+\left(x'\right)^{2}\right]}{\left[1+\left(x'\right)^{2}\right]^{2}}\, dy, \tag{10}$$

In equations (9) and (10), the dashes represent derivative with respect to x and y respectively. Where the suffix $\perp$ has been placed to designate the force due to the external flow along the *y*-axis, the transverse direction.

The proposed conjecture is, of, course, subject to restrictions on the geometry of the meridional body profile $y(x)$ a of continuously turning tangent implying that $y'(x)$ is continuous together with $y''(x) \neq 0$, thereby avoiding corners or sharp edges or other kind of nodes and straight line portions, $y = ax + b,\ \ x_1 \leq x \leq x_2$. Also, it should be noted here that the method holds good for convex axially symmetric bodies which possesses fore-aft symmetry about the equatorial axis perpendicular to the axis of symmetry (polar axis). Apart from this argument, It is interesting to note here that the proposed conjecture is applicable also to those axi-symmetric bodies which fulfills the condition of continuously turning tangent but does not possesses fore-aft symmetry like egg shaped body [Datta & Srivastava (1999)]. This conjecture is much simpler to evaluate the numerical values of drag than other existing numerical methods like Boundary Element Method (BEM), Finite Element Analysis (FEA) as it can be applied to a large set of convex axi-symmetric bodies possessing fore-aft symmetry about maximal radius situated in the middle of the body for which analytical solution is not available or impossible to evaluate.

Since both axial and transverse flows have been considered in a free stream results of the force at an oblique angle of attack may be resolved into its components to get the required result. The present analysis can be extended to generate a drag formula for axi- symmetric bodies for more complex flows like paraboloidal flow for which free stream may be represented by average velocity [Chwang and Wu, part 2, (1975)]. Many authors are now working in this direction and also searching the avenues of this analysis for the non-linear Stokes flow.

The proposed analysis can be extended to calculate the couple on a body rotating about its axis of symmetry and about an axis perpendicular to the axis of symmetry. Many authors are working on it and the corresponding studies in this direction may soon appear. .

3. Formulation of the Problem

Consider the axially symmetric body defined by

$$r = a\left[1 + \varepsilon \sum_{k=0}^{\infty} d_k P_k\left(\mu\right)\right], \quad \mu = \cos\theta, \tag{11}$$

where $(r,\ \theta)$ are spherical polar coordinates, ε is the small deformation parameter, d_k's are the design or shape factors and $P_k\ (\mu)$ are Legendre function of first kind. For small values of parameter 'ε', equation (11) represents the deformed sphere.

Now, our prime task is to calculate the perturbation expressions of drag on the deformed sphere placed in quadratic and stagnation like quadratic flow in both axial and transverse situations corrected up to the order of $O(\varepsilon^2)$ with the help of proposed conjecture in the Section 2 and the concept of surface average velocity on body surface proposed by Chwang and Wu (1975, part 2 and 3).

4. Solution

4.1. Axial Quadratic Flow

We consider a axi-symmetric body equation (11) placed in an axial flow with a paraboloidal velocity profile

$$U_{paraboloid} = K\,(y^2 + z^2), \tag{12}$$

or

$$U_{paraboloid} = (\alpha y^2 + z^2), \tag{13}$$

along axis of symmetry which is x-axis, where K is arbitrary constant and α is also arbitrary constant related to flow may be positive or negative indicating that paraboloidal flow is either elliptic or hyperbolic. When α vanishes, the paraboloidal flow degenerates into a two-dimensional parabolic flow. For arbitrary positive values of α, it represents Hagen-Poiseuille flow through a pipe of elliptic cross-section. If $\alpha = 1$, it becomes a paraboloidal flow of revolution which corresponds to Heigen-Poiseuille flow in a circular tube. Hyperbolic paraboloidal flow ($\alpha < 0$) may not exist physically, but it can certainly serve as a local component of a more complicated flow field.

By DS-conjecture (Datta and Srivastava, 1999), the expression of drag on axially symmetric body placed in axial uniform stream U (from equation (2) and (7)) is

$$F_x = \frac{1}{2}\frac{\lambda b^2}{h_x}, \quad \text{where} \quad \lambda = 6\pi\mu U_x \tag{14}$$

and

$$h_x = \left(\frac{3}{8}\right)\int_0^{\pi} R\sin^3\,\alpha\; d\alpha, \tag{15}$$

where the suffix 'x' has been introduced to assert that the force is in the axial direction. Now, equation (12) may be re-written as

$$F_x = \frac{1}{2}\frac{(6\pi\mu)b^2}{h_x}U_x = \frac{3\pi\mu b^2}{h_x}U_e, \tag{16}$$

where U_e is precisely the surface average of the primary flow velocity

$$U = Kr^2 = K\,(y^2 + z^2), \tag{17}$$

over a surface of axi-symmetric body (conjectured by Chwang and Wu, 1975, part 2), '$b=(r)_{\theta=\pi/2}$' is largest cross-section radius situated at the mid of the body, and h_x is given by equation (13). These values may also be written in perturbation form [see Srivastava et al. (2012)] as

$$b^2 = a^2 \left[\begin{array}{l} 1+2\varepsilon\left(d_0 - \frac{1}{2}d_2 + \frac{3}{8}d_4 \cdots\right) \\ +\varepsilon^2 \left\{ \begin{array}{l} \left(d_0{}^2 + \frac{1}{4}d_2{}^2 + \frac{9}{64}d_4{}^2 \cdots\right) \\ +2d_0\left(-\frac{1}{2}d_2 + \frac{3}{8}d_4 \cdots\right) + 2d_2\left(\frac{3}{8}d_4 \cdots\right) \end{array} \right\} \end{array} \right], \tag{18}$$

and

$$h_x = \frac{a}{2}\left[1+\varepsilon\left(d_0 - \frac{4}{5}d_2\right)\right]. \tag{19}$$

Now, finally, by utilizing the equations (18) and (19), the expression for axial Stokes drag [given in equation (16)] experienced by body (11) placed in parabolic flow comes out to be

$$F_x = 6\pi\mu a \left[\begin{array}{l} 1+\varepsilon\left\{d_0 - \frac{1}{5}d_2 + \frac{3}{8}d_4 \cdots\right\} \\ +\varepsilon^2 \left\{ \begin{array}{l} \left(2d_0{}^2 + \frac{89}{100}d_2{}^2 + \frac{9}{64}d_4{}^2 + \cdots\right) \\ +2d_0\left(-\frac{21}{10}d_2 + \frac{3}{8}d_4 + \cdots\right) + 2d_2\left(\frac{3}{8}d_4 + \cdots\right) + \cdots \end{array} \right\} + O\left(\varepsilon^3\right) \end{array} \right] U_e. \tag{20}$$

4.2. Transverse Quadratic Flow

We consider an axi-symmetric body equation (11) placed under a flow with a paraboloidal velocity profile

$$U_{paraboloid} = K\,(x^2 + z^2), \tag{21}$$

or

$$U_{paraboloid} = (\beta x^2 + z^2), \tag{22}$$

along axis of symmetry which is y-axis, where K and β are arbitrary constant.

The expression of the Stokes drag on such type of axially symmetric bodies placed in transverse flow (uniform flow perpendicular to the axis of symmetry) is given by (Datta and Srivastava, 1999)

$$F_y = \left(\frac{1}{2}\right)\frac{\lambda b^2}{h_y}, \quad \text{where } \lambda = 6\pi\mu U_y, \tag{23}$$

and

$$h_y = \left(\frac{3}{16}\right)\int_0^{\pi} R\left(2\ \sin\ \alpha - \sin^3\ \alpha\right)\, d\alpha. \tag{24}$$

Now, equation (23) may be re-written as

$$F_y = \frac{1}{2}\frac{\left(6\pi\mu\right)b^2}{h_y}U_y = \frac{3\pi\mu b^2}{h_y}U_e, \tag{25}$$

where U_e is precisely the surface average of the primary flow velocity

$$U = Kr^2 = K\,(x^2 + z^2), \tag{26}$$

over a surface of axi-symmetric body (defined by Chwang and Wu, 1975, part 2), 'b' is largest cross-section radius situated at the mid of the body, and h_y is given by equation (24). These values may also be written in perturbation form [see Srivastava et al. (2012)] as

$$b^2 = a^2\left[\begin{array}{l} 1 + 2\varepsilon\left(d_0 - \frac{1}{2}d_2 + \frac{3}{8}d_4\cdots\right) \\ \quad + \varepsilon^2\left\{\begin{array}{l}\left(d_0{}^2 + \frac{1}{4}d_2{}^2 + \frac{9}{64}d_4{}^2\cdots\right) \\ + 2d_0\left(-\frac{1}{2}d_2 + \frac{3}{8}d_4\cdots\right) + 2d_2\left(\frac{3}{8}d_4\cdots\right)\end{array}\right\}\end{array}\right], \tag{27}$$

and

$$h_y = \frac{a}{2}\left[1 + \varepsilon\left(d_0 - \frac{11}{10}d_2 - \frac{3}{2}d_4 - \frac{3}{2}d_6 - \cdots\right)\right]. \tag{28}$$

Now, finally, by utilizing the equations (27) and (28), the expression for transverse Stokes drag [given in equation (25)] experienced by body equation (11) placed in parabolic flow comes out to be

$$F_y = 6\pi\mu a \left[\begin{array}{l} 1+\varepsilon\left(d_0+\frac{1}{10}d_2+\frac{9}{4}d_4+\cdots\right) \\ +\varepsilon^2\left\{ \begin{array}{r} \left(\frac{9}{25}d_2^{\ 2}+\frac{225}{64}d_4^{\ 2}+\cdots\right)+2d_0\left(-\frac{1}{2}d_2+\frac{3}{8}d_4+...\right) \\ +2d_2\left(\frac{51}{40}d_4+...\right) \end{array} \right\}+O\left(\varepsilon^3\right) \end{array} \right] U_e. \tag{29}$$

4.3. Axial Stagnation Like Quadratic Flow

We consider an axi-symmetric body placed in a longitudinal stagnation-like quadratic flow with velocity

$$\boldsymbol{U} = x^2\ e_x - 2xy\ e_y, \tag{30}$$

and

$$U = |U| = (x^4 + 4x^2y^2)^{1/2}, \tag{31}$$

along axis of symmetry which is x-axis, which obviously satisfies the Stokes equation (1) if the pressure associated with it is $2\mu x$. The stagnation plane is the centre-plane $x = 0$. In the half-space $x < 0$ the flow is towards the stagnation plane while in the half-space $x > 0$ it is away from this plane. This type of quadratic flow is important since it can serve as a component in the general study of the motion of a spheroidal particle placed in a paraboloidal flow whose direction does not coincide with any one of the principal axes of the spheroid.

By DS-conjecture [Datta and Srivastava (1999)], the expression of the drag on axially symmetric body placed in axial uniform stream U (from equations (2) and (3)) is

$$F_x = \frac{1}{2}\frac{\lambda b^2}{h_x},\ \ \text{where}\ \ \lambda = 6\pi\mu U_x \tag{32}$$

and

$$h_x = \left(\frac{3}{8}\right)\int_0^{\pi} R\ \sin^3\ \alpha\ d\alpha, \tag{33}$$

where the suffix ‘x’ has been introduced to assert that the force is in the axial direction. Now, equation (32) may be rewritten as

$$F_x = \frac{1}{2}\frac{(6\pi\mu)b^2}{h_x}U_x = \frac{3\pi\mu b^2}{h_x}U_e, \tag{34}$$

where U_e is precisely the surface average of the primary flow velocity

$$U = Kr^2 = K(y^2 + z^2), \tag{35}$$

over a surface of axi-symmetric body (conjectured by Chwang and Wu, 1975, part 2), 'b' is largest cross-section radius situated at the mid of the body, and h_x is given by equation (33). These values may also be written in perturbation form [see Srivastava et al. (2012)] as

$$b^2 = a^2 \left[\begin{array}{l} 1 + 2\varepsilon\left(d_0 - \frac{1}{2}d_2 + \frac{3}{8}d_4 \cdots\right) \\ + \varepsilon^2 \left\{ \begin{array}{r} \left(d_0{}^2 + \frac{1}{4}d_2{}^2 + \frac{9}{64}d_4{}^2 \cdots\right) + 2d_0\left(-\frac{1}{2}d_2 + \frac{3}{8}d_4 \cdots\right) \\ + 2d_2\left(\frac{3}{8}d_4 \cdots\right) \end{array} \right\} \end{array} \right], \tag{36}$$

and

$$h_x = \frac{a}{2}\left[1 + \varepsilon\left(d_0 - \frac{4}{5}d_2\right)\right]. \tag{37}$$

Now, finally, by utilizing the equations (18) and (19), the expression for axial Stokes drag [given in equation (34)] experienced by body equation (11) placed in parabolic flow comes out to be

$$F_x = 6\pi\mu a \left[\begin{array}{l} 1 + \varepsilon\left\{d_0 - \frac{1}{5}d_2 + \frac{3}{8}d_4 \cdots\right\} \\ + \varepsilon^2 \left\{ \begin{array}{r} \left(2d_0{}^2 + \frac{89}{100}d_2{}^2 + \frac{9}{64}d_4{}^2 + \cdots\right) \\ + 2d_0\left(-\frac{21}{10}d_2 + \frac{3}{8}d_4 + \cdots\right) \\ + 2d_2\left(\frac{3}{8}d_4 + \cdots\right) + \cdots \end{array} \right\} + O\left(\varepsilon^3\right) \end{array} \right] U_e \,. \tag{38}$$

4.4. Transverse Stagnation Like Paraboloid Flow

We consider an axi-symmetric body placed in a longitudinal stagnation-like quadratic flow with velocity

$$\boldsymbol{U} = 2xy\, e_x - y^2\, e_y \tag{39}$$

$$U = |U| = (y^4 + 4x^2y^2)^{1/2}, \tag{40}$$

along axis of symmetry which is y-axis.

The expression of Stokes drag on such type of axially symmetric bodies placed in transverse flow (uniform flow perpendicular to the axis of symmetry) is given by [Datta and Srivastava (1999)]

$$F_y = \left(\frac{1}{2}\right)\frac{\lambda b^2}{h_y}, \quad \text{where } \lambda = 6\pi\mu U_y\ , \tag{41}$$

and

$$h_y = \left(\frac{3}{16}\right)\int_0^{\pi} R\left(2\ \sin\ \alpha - \ \sin^3\ \alpha\right)\, d\alpha\ . \tag{42}$$

Now, equation (41) may be re-written as

$$F_y = \frac{1}{2}\frac{(6\pi\mu)b^2}{h_y}U_y = \frac{3\pi\mu b^2}{h_y}U_e, \tag{43}$$

where U_e is precisely the surface average of the primary flow velocity

$$U = Kr^2 = K\,(x^2 + z^2), \tag{44}$$

over a surface of axi-symmetric body (conjectured by Chwang and Wu, 1975, part 2), 'b' is largest cross-section radius situated at the mid of the body, and h_y is given by equation (42). These values may also be written in perturbation form [see Srivastava et al. (2012)] as

$$b^2 = a^2\left[\begin{array}{l} 1 + 2\varepsilon\left(d_0 - \frac{1}{2}d_2 + \frac{3}{8}d_4\cdots\right) \\ +\varepsilon^2\left\{\begin{array}{l}\left(d_0{}^2 + \frac{1}{4}d_2{}^2 + \frac{9}{64}d_4{}^2\cdots\right) \\ +2d_0\left(-\frac{1}{2}d_2 + \frac{3}{8}d_4\cdots\right) + 2d_2\left(\frac{3}{8}d_4\cdots\right)\end{array}\right\}\end{array}\right], \tag{45}$$

and

$$h_y = \frac{a}{2}\left[1+\varepsilon\left(d_0 - \frac{11}{10}d_2 - \frac{3}{2}d_4 - \frac{3}{2}d_6 - \cdots\right)\right]. \tag{46}$$

Now, finally, by utilizing the equations (45) and (46), the expression for transverse Stokes drag [given in equation (43)] experienced by body, equation (11) placed in parabolic flow comes out to be

$$F_y = 6\pi\mu a\left[\begin{array}{l} 1+\varepsilon\left(d_0 + \frac{1}{10}d_2 + \frac{9}{4}d_4 + \cdots\right) \\ +\varepsilon^2\left\{\begin{array}{l}\left(\frac{9}{25}d_2^{\,2} + \frac{225}{64}d_4^{\,2} + \cdots\right) \\ +2d_0\left(-\frac{1}{2}d_2 + \frac{3}{8}d_4 + \ldots\right) + 2d_2\left(\frac{51}{40}d_4 + \ldots\right)\end{array}\right\} + O\left(\varepsilon^3\right)\end{array}\right]U_e\ . \tag{47}$$

Now, we apply the expressions (20), (29) and (38), (47) to evaluate perturbation forms of drag on axially symmetric bodies like sphere, spheroid (prolate and oblate) and egg-shaped body placed under longitudinal and transverse unbounded quadratic as well as axial and transverse stagnation like quadratic flows.

5. Sphere

5.1. Axial Paraboloidal Flow

We consider a sphere having radius ‘a’ placed under axial paraboloidal flow velocity

$$U_{paraboloid} = K\,(y^2 + z^2),$$

by considering the surface average velocity $U_e = (2/3)Ka^2$, over a spherical surface $r = a$, obtained by taking $\varepsilon = 0$ in equation (11), the value of ‘b’ is equal to ‘a’ (from equation (18)), the value of $h_x = a/2$ (from equation (19)), the expression of drag on sphere comes out to be (with the use of equation (20))

$$F_x = \frac{3\pi\mu b^2}{h_x}U_e = 6\pi\mu a\left(1\right)\left(\frac{2}{3}Ka^2\right) = 4\pi\mu Ka^3, \tag{48}$$

which matches that obtained by Chwang and Wu (1975, part 2, p. 807, equation 69) treated with singularity method. This expression directly reduces to $4\pi\mu a^3$ for $K = 1$.

5.2. Transverse Flow

We consider a sphere having radius ‘a’ placed under transverse paraboloidal flow velocity

$U_{paraboloid} = K\,(x^2 + z^2)$,

by considering the surface average velocity $U_e = (2/3)Ka^2$, over a spherical surface $r = a$, obtained by taking $\varepsilon = 0$ in equation (11), the value of 'b' is equal to 'a' (from equation (27)), the value of $h_y = a/2$ (from equation (28)), the expression of drag on sphere comes out to be (with the use of equation (29))

$$F_y = \frac{3\pi\mu b^2}{h_y} U_e = 6\pi\mu a(1)\left(\frac{2}{3}Ka^2\right) = 4\pi\mu K a^3, \tag{49}$$

which is exactly the same as obtained in axial situation (48). The reason behind it is the fore and aft symmetry of the spherical body. This expression straight forward reduces to $4\pi\mu a^3$ for $K = 1$.

5.3. Axial Stagnation Like Quadratic Flow

We consider a sphere having radius 'a' placed under axial stagnation like paraboloidal flow having velocity $U = |U| = (x^4 + 4x^2y^2)$, by considering the surface average velocity $U_e = (1/3)a^2$, over a spherical surface $r = a$, obtained by taking $\varepsilon = 0$ in equation (11), the value of 'b' is equal to 'a' (from equation (36)), the value of $h_x = a/2$ (from equation (37)), the expression of drag on sphere comes out to be (with the use of equation (38))

$$F_x = \frac{3\pi\mu a^2}{h_x} U_e = 6\pi\mu a(1)\left(\frac{1}{3}a^2\right) = 2\pi\mu a^3, \tag{50}$$

which matches with that obtained by Chwang and Wu (1975, part 3, p. 26, equation 34) treated with singularity method.

5.4. Transverse Stagnation Like Quadratic Flow

We consider a sphere having radius 'a' placed under transverse stagnation like paraboloidal flow velocity $U = |U| = (y^4 + 4x^2y^2)^{1/2}$, by considering the surface average velocity $U_e = (1/3)a^2$, over a spherical surface $r = a$, obtained by taking $\varepsilon = 0$ in equation (11), the value of 'b' is equal to 'a' (from equation (45)), the value of $h_y = a/2$ (from equation (46)), the expression of drag on sphere in transverse stagnation like paraboloidal flow comes out to be (with the use of equation (47))

$$F_y = \frac{3\pi\mu a^2}{h_y} U_e = 6\pi\mu a(1)\left(\frac{1}{3}a^2\right) = 2\pi\mu a^3, \tag{51}$$

which matches with that obtained by Chwang and Wu (1975, part 3, p. 28, eq. 42) treated with singularity method and is exactly the same as obtained in axial situation (10.1). The reason behind it is the fore and aft symmetry of the spherical body.

6. Prolate Spheroid

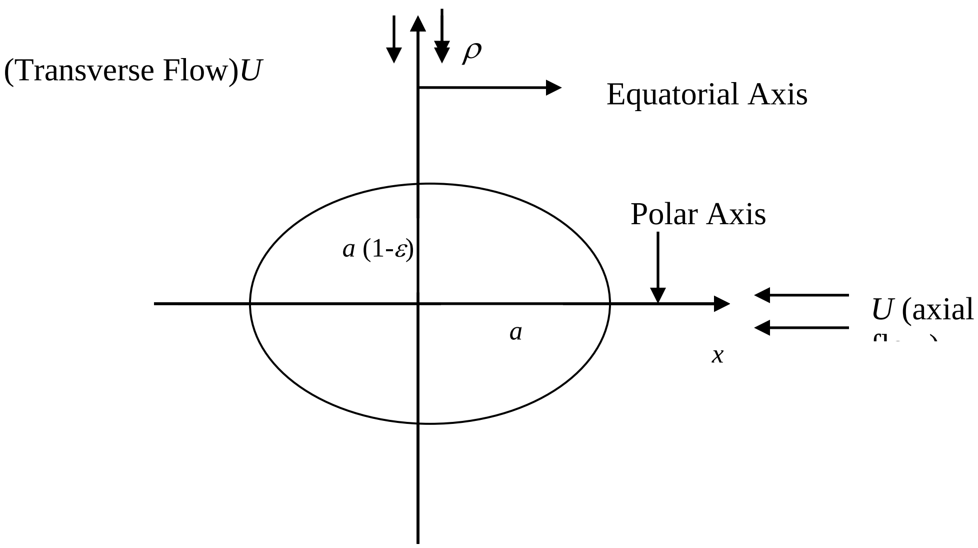

Figure 2. Prolate spheroid in meridional two-dimensional plane (ρ, x)

We consider the prolate spheroid, as it belongs to the class of axi-symmetric deformed sphere, whose Cartesian equation is

$$\frac{y^2+z^2}{a^2(1-\varepsilon)^2}+\frac{x^2}{a^2}=1, \tag{52}$$

where equatorial radius is a $(1-\varepsilon)$ and polar radius is a, in which deformation parameter ε is positive and sufficiently small that squares and higher powers of it may be neglected. Its polar equation [up to the order of $O(\varepsilon)$], by using the transformation,

$$x = r\cos\theta,\ \rho = r\sin\theta, \tag{53}$$

is

$$r = a\,(1-\varepsilon\sin^2\theta)\,. \tag{54}$$

It can be written in linear combination of Legendre functions of first kind [Happel and Brenner (1964), Senchenko and Keh (2006)]

$$r = a\left[1+\varepsilon\ \left\{-\frac{2}{3}P_0(\mu)+\frac{2}{3}P_2(\mu)\right\}\right],\quad \mu=\cos\ \theta, \tag{55}$$

On comparing this equation with the polar equation of deformed sphere (11), the appropriate design factors for this prolate spheroid are

$$d_0 = -\frac{2}{3},\ \ d_1 = 0,\ \ d_2 = \frac{2}{3},\ \ d_k = 0,\ \ \text{for } k \geq 3, \tag{56}$$

with

$$P_0(\mu) = 1,\ \ P_1(\mu) = \mu \ \text{ and } \ P_2(\mu) = \frac{3\mu^2 - 1}{2},\ \ \mu = \cos\ \theta. \tag{57}$$

Also, this polar equation (54) of prolate spheroid may be written in terms of Gegenbauer functions of first kind by using the following well known relation

$$\mathcal{J}_k(\mu) = \frac{P_{k\text{-}2}(\mu) - P_k(\mu)}{2k\text{-}1},\ \ k \geq 2,\ \ \mu = \cos\ \theta, \tag{58}$$

as

$$r = a\left[1 - 2\varepsilon\ \mathcal{J}_2(\mu)\right],\quad \mu = \cos\theta. \tag{59}$$

Now, we find the expression of Stokes drag on this axi-symmetric prolate body placed in quadratic and stagnation like quadratic flow in both longitudinal and transverse (in which uniform stream is parallel and perpendicular to polar axis or axis of symmetry) situations with the aid of general expressions of drag [equations (20), (29) and (38), (47)].

6.1. Axial Parabolic Flow

For $\alpha \neq 1$, surface average velocity U_e in perturbed form for the surface [equation (55)] is

$$\begin{aligned} U_e &= \frac{a^2}{3}(1+\alpha)\left(1 - e^2\right) \\ &= \frac{a^2}{3}(1+\alpha)(1-\varepsilon)^2,\ \ \left[\text{by using}\ \ \frac{b^2}{a^2} = 1 - e^2 = (1-\varepsilon)^2\right] \\ &= \frac{a^2}{3}(1+\alpha)\left(1 - 2\varepsilon + \varepsilon^2\right), \end{aligned} \tag{60}$$

the value of b^2, for body surface (55), from equation (18) and (56), in perturbed form, is as it should be, due to the fact that $b = a\ (1-\varepsilon)$, for prolate perturbed spheroid (see figure 2), the value of h_x, for the body surface [equation (55)], from equation (19), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right]= a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{61}$$

$$\begin{aligned} h_x &= \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{4}{5}d_2\right)\right] \\ &= \frac{a}{2}\left[1-\frac{6}{5}\varepsilon\right]. \end{aligned} \tag{62}$$

Now, for flow constant $\alpha \neq 1$, the expression of drag, F_x , from equation (20), by utilizing (55) and (60), comes out to be

$$\begin{aligned} F_x &= 6\pi\mu a\left[1+\varepsilon\left\{d_0-\frac{1}{5}d_2\right\}+\varepsilon^2\left\{2d_0{}^2+\frac{89}{100}d_2{}^2-\frac{21}{5}d_0d_2\right\}\right]U_e \\ &= 2\pi\mu a^3\left(1+\alpha\right)\left[1-\frac{14}{5}\varepsilon+\frac{1294}{225}\varepsilon^2+....\right]. \end{aligned} \tag{63}$$

For $\alpha=1$, equation (63) reduces into the form

$$F_x = 4\pi\mu a^3\left[1-\frac{14}{5}\varepsilon+\frac{1294}{225}\varepsilon^2+....\right], \tag{64}$$

which further reduces to $4\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in axial quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$.

If we consider the polar equation of prolate spheroid involving second order perturbation terms [see Chang and Keh, 2009] as

$$r = a\left[1-\varepsilon\ \sin^2\theta-\frac{3}{2}\varepsilon^2\ \cos^2\theta\ \sin^2\theta\right], \tag{65}$$

then, for $\alpha \neq 1$, independent application of DS conjecture [Datta and Srivastava, 1999] on surface (65) provide the axial Stokes drag as

$$\begin{aligned} F_x &= 6\pi\mu a\left[1-\frac{4}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]U_e \\ &= 6\pi\mu a\left[1-\frac{4}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}(1+\alpha)(1-\varepsilon)^2\right] \\ &= 2\pi\mu a^3(1+\alpha)\left[1-\frac{14}{5}\varepsilon+\frac{457}{175}\varepsilon^2+\cdots\right], \end{aligned} \tag{66}$$

for α =1, this expression reduces to the form

$$F_x = 4\pi\mu a^3\left[1-\frac{14}{5}\varepsilon+\frac{457}{175}\varepsilon^2+\cdots\right], \tag{67a}$$

which immediately reduces to $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial parabolic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag[(64) and (67a)] are in disagreement because initially the polar equation [(11), (55)] contains only first order terms in deformation parameter 'ε' while in later equation (65), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (67a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fx} by normalizing the drag (67a) by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial quadratic flow, as

$$C_{F_x} = \frac{F_x}{4\pi\mu a^3} = \left[1-\frac{14}{5}\varepsilon+\frac{457}{175}\varepsilon^2+\cdots\right]. \tag{67b}$$

6.2. Transverse Parabolic Flow

For $\beta \neq 1$, surface average velocity U_e in perturbed form for the surface [equation (55)] is

$$\begin{aligned} U_e &= \frac{a^2}{3}\left(1+\beta-e^2\right) \\ &= \frac{a^2}{3}\left[\beta+(1-\varepsilon)^2\right], \quad \left[\text{by using } \frac{b^2}{a^2}=1-e^2=(1-\varepsilon)^2\right] \\ &= \frac{a^2}{3}\left(1+\beta-2\varepsilon+\varepsilon^2\right), \end{aligned} \tag{68}$$

the value of b^2, for body surface [equation (55)], from equation (18) and (56), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right]$$
$$= a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{69}$$

for prolate perturbed spheroid (see Figure 2), the value of h_y, for body surface (55), from equation (28), in perturbed form, is

$$h_y = \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{11}{10}d_2\right)\right]$$
$$= \frac{a}{2}\left[1-\frac{7}{5}\varepsilon\right]. \tag{70}$$

Now, for flow constant $\beta\neq1$, the expression of drag, F_y , from equation (29), by utilizing equations (55) and (68), comes out to be

$$F_y = 6\pi\mu a\left[1+\varepsilon\left\{d_0+\frac{1}{10}d_2\right\}+\varepsilon^2\left\{\frac{9}{25}d_2{}^2-d_0d_2\right\}+\cdots\right]U_e$$
$$= 2\pi\mu a^3\left(1+\beta-2\varepsilon+\varepsilon^2\right)\left[1-\frac{3}{5}\varepsilon+\frac{136}{225}\varepsilon^2+\cdots\right],$$
$$= 2\pi\mu a^3\left[\left(1+\beta\right)-\frac{\left(13+3\beta\right)}{5}\varepsilon+\frac{\left(631+136\beta\right)}{225}\varepsilon^2+\cdots\right]. \tag{71}$$

For β =1, equation (71) reduces into the form

$$F_y = 4\pi\mu a^3\left[1-\frac{8}{5}\varepsilon+\frac{767}{550}\varepsilon^2+\cdots\right], \tag{72}$$

which further reduces to $4\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in transverse quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon\to0$.

If we consider the polar equation of prolate spheroid involving second order perturbation terms [see Chang and Keh (2009)] as

$$r = a\left[1-\varepsilon\sin^2\theta-\frac{3}{2}\varepsilon^2\cos^2\theta\sin^2\theta\right], \tag{73}$$

then, for $\beta\neq1$, independent application of DS conjecture [Datta and Srivastava (1999)] on surface (65) provide the transverse Stokes drag as

$$F_y = 6\pi\mu a\left[1-\frac{3}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right]U_e$$
$$= 6\pi\mu a\left[1-\frac{3}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}\left[\beta+(1-\varepsilon)^2\right]\right] \tag{74}$$
$$= 2\pi\mu a^3\left[(1+\beta)-\frac{(13+3\beta)}{5}\varepsilon+\frac{(761-9\beta)}{350}\varepsilon^2+\cdots\right],$$

for β=1, this expression reduces to the form

$$F_y = 4\pi\mu a^3\left[1-\frac{8}{5}\varepsilon+\frac{188}{175}\varepsilon^2+\cdots\right], \tag{75a}$$

which immediately reduces to $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse parabolic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag[equations (72) and (75a)] are in disagreement because initially the polar equation [equations (11) and (55)] contains only first order terms in deformation parameter 'ε' while in later equation (73), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (75a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fx} by normalizing the drag (75a) by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse quadratic flow, as

$$C_{F_y} = \frac{F_y}{4\pi\mu a^3} = \left[1-\frac{8}{5}\varepsilon+\frac{188}{175}\varepsilon^2+\cdots\right]. \tag{75b}$$

6.3. Axial Stagnation Like Quadratic Flow

The surface average velocity U_e in perturbed form for the surface [equation (55)] is

$$U_e = \frac{a^2}{3}, \tag{76}$$

the value of b^2, for body surface [equation (55)], from equation (18) and (56), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right] \tag{77}$$
$$= a^2\left[1-2\varepsilon+\varepsilon^2\right],$$

as it should be, for prolate perturbed spheroid (see Figure 2), the value of h_x, for body surface [equation (55)], from equation (19), in perturbed form, is

$$h_x = \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{4}{5}d_2\right)\right]$$
$$= \frac{a}{2}\left[1-\frac{6}{5}\varepsilon\right]. \tag{78}$$

Now, the expression of drag, F_x, from equation (20), by utilizing equations (55) and (76), comes out to be

$$F_x = 6\pi\mu a\left[1+\varepsilon\left\{d_0-\frac{1}{5}d_2\right\}+\varepsilon^2\left\{2d_0{}^2+\frac{89}{100}d_2{}^2-\frac{21}{5}d_0d_2\right\}\right]U_e \tag{79}$$
$$= 2\pi\mu a^3\left[1-\frac{4}{5}\varepsilon+\frac{709}{225}\varepsilon^2+\cdots\right].$$

which further reduces to $2\pi\mu a^3$, the classical drag for sphere having radius '*a*' placed in axial stagnation like quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$.

If we consider the polar equation of prolate spheroid involving second order perturbation terms [see Chang and Keh (2009)] as independent application of DS conjecture [Datta and Srivastava (1999)] on surface [equation (80)] provide the axial Stokes drag as

$$r = a\left[1-\varepsilon\sin^2\theta-\frac{3}{2}\varepsilon^2\cos^2\theta\sin^2\theta\right], \tag{80}$$

$$F_x = 6\pi\mu a\left[1-\frac{4}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]U_e$$
$$= 6\pi\mu a\left[1-\frac{4}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}\right]$$
$$= 2\pi\mu a^3\left[1-\frac{4}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right], \tag{81a}$$

which immediately reduces to $2\pi\mu a^3$, the drag on sphere having radius '*a*', placed in axial stagnation like quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag[equations (79) and (81a)] are in disagreement because initially the polar equation [equations (11) and (55)] contains only first order terms in deformation parameter 'ε' while in later equation (80), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (81a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fx} by normalizing the drag (81a) by $2\pi\mu a^3$, the drag on sphere having radius '*a*', placed in axial stagnation like quadratic flow, as

$$C_{F_x} = \frac{F_x}{2\pi\mu a^3} = \left[1 - \frac{4}{5}\varepsilon + \frac{2}{175}\varepsilon^2 + \cdots\right]. \tag{81b}$$

6.4. Transverse Stagnation Like Quadratic Flow

The surface average velocity U_e in perturbed form for the surface [equation (55)] is

$$\begin{aligned} U_e &= \frac{b^2}{3} = \frac{a^2}{3}\left(1-e^2\right) \\ &= \frac{a^2}{3}\left[\left(1-\varepsilon\right)^2\right], \qquad \left[\text{by using } \frac{b^2}{a^2} = 1-e^2 = \left(1-\varepsilon\right)^2\right] \\ &= \frac{a^2}{3}\left(1-2\varepsilon+\varepsilon^2\right), \end{aligned} \tag{82}$$

the value of b^2, for body surface [equation (55)], from equations (18) and (56), in perturbed form, is

$$b^2 = a^2\left[1 + 2\varepsilon\left(d_0 - \frac{1}{2}d_2\right) + \varepsilon^2\left(d_0{}^2 + \frac{1}{4}d_2{}^2 - d_0 d_2\right)\right] = a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{83}$$

for prolate perturbed spheroid (see figure 2), the value of h_y, for body surface [equation (55)], from equation (28), in perturbed form, is

$$h_y = \frac{a}{2}\left[1 + \varepsilon\left(d_0 - \frac{11}{10}d_2\right)\right] = \frac{a}{2}\left[1 - \frac{7}{5}\varepsilon\right]. \tag{84}$$

Now, for flow constant $\beta \neq 1$, the expression of drag, F_y , from equation (29), by utilizing [equations (55) and (68), comes out to be

$$\begin{aligned} F_y &= 6\pi\mu a\left[1 + \varepsilon\left\{d_0 + \frac{1}{10}d_2\right\} + \varepsilon^2\left\{\frac{9}{25}d_2{}^2 - d_0 d_2\right\} + \cdots\right]U_e \\ &= 2\pi\mu a^3\left(1-2\varepsilon+\varepsilon^2\right)\left[1 - \frac{3}{5}\varepsilon + \frac{136}{225}\varepsilon^2 + \cdots\right], \\ &= 2\pi\mu a^3\left[1 - \frac{13}{5}\varepsilon + \frac{631}{225}\varepsilon^2 + \cdots\right], \end{aligned} \tag{85}$$

which further reduces to $2\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in transverse quadratic stagnation like flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$.

If we consider the polar equation of prolate spheroid involving second order perturbation terms [see Chang and Keh (2009)] as

$$r = a\left[1 - \varepsilon \sin^2\theta - \frac{3}{2}\varepsilon^2 \cos^2\theta \sin^2\theta\right], \tag{86}$$

independent application of DS conjecture [Datta and Srivastava (1999)] on surface [equation (86)] provide the transverse Stokes drag as

$$\begin{aligned} F_y &= 6\pi\mu a\left[1 - \frac{3}{5}\varepsilon - \frac{9}{350}\varepsilon^2 \cdots\right]U_e \\ &= 6\pi\mu a\left[1 - \frac{3}{5}\varepsilon - \frac{9}{350}\varepsilon^2 \cdots\right]\left[\frac{a^2}{3}\left[(1-\varepsilon)^2\right]\right] \\ &= 2\pi\mu a^3\left[1 - \frac{13}{5}\varepsilon + \frac{761}{350}\varepsilon^2 + \cdots\right], \end{aligned} \tag{87a}$$

which immediately reduces to $2\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse stagnation like quadratic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag[equations (85) and (87a)] are in disagreement because initially the polar equation [equations (11) and (55)] contains only first order terms in deformation parameter 'ε' while in later equation (73), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (87a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag

$$C_{F_y} = \frac{F_y}{2\pi\mu a^3} = \left[1 - \frac{13}{5}\varepsilon + \frac{761}{350}\varepsilon^2 + \cdots\right]. \tag{87b}$$

coefficient C_{Fy} by normalizing the drag (87a) by $2\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse stagnation like quadratic flow, as

7. Oblate Spheroid

We consider the oblate spheroid, as it belongs to the class of axi-symmetric deformed sphere whose Cartesian equation is

$$\frac{y^2 + z^2}{a^2} + \frac{x^2}{a^2(1-\varepsilon)^2} = 1, \tag{88}$$

where equatorial radius is 'a' and polar radius is $a\,(1-\varepsilon)$, in which deformation parameter 'ε' is positive and sufficiently small that squares and higher powers of it may be neglected. Its polar equation, [by using $x = r\cos\theta$, $\rho = r\sin\theta$, $\rho = (y^2 + z^2)^{1/2}$, up to the order of $O(\varepsilon)$] is

$$r = a\,(1 - \varepsilon \cos^2 \theta)\,, \tag{89}$$

it can be written in linear combination of Legendre functions of first kind [Happel and Brenner, 1964, Senchenko and Keh, 2009]

$$r = a\left[1 - \varepsilon\left\{\frac{1}{3}P_0(\mu) + \frac{2}{3}P_2(\mu)\right\}\right],\ \ \mu = \cos\theta. \tag{90}$$

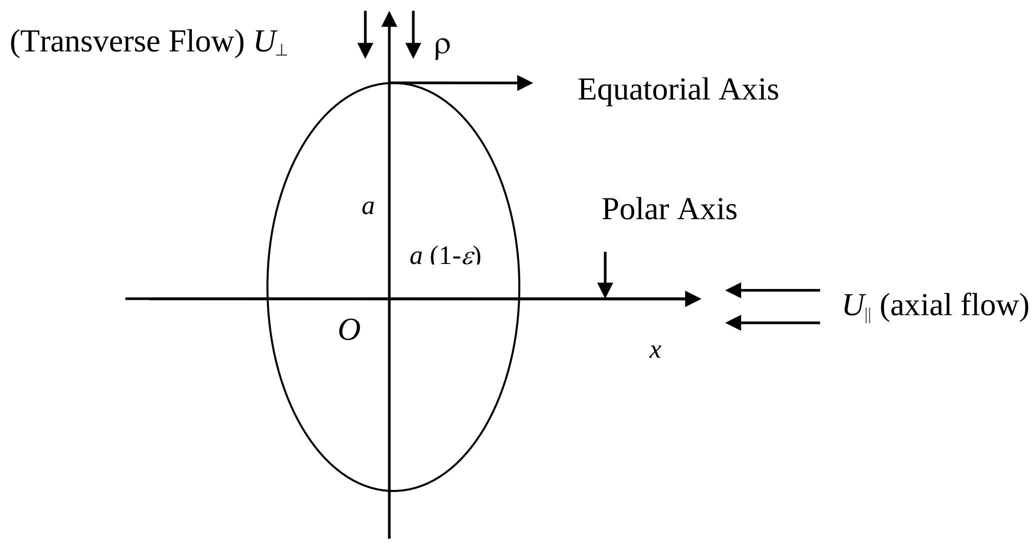

Figure 3. Oblate spheroid in meridional two-dimensional plane (x, ρ)

On comparing this equation with the polar equation of deformed sphere [equation (11)], the appropriate design factors for this oblate spheroid are

$$d_0 = -\frac{1}{3},\ \ d_1 = 0,\ \ d_2 = -\frac{2}{3},\ \ d_k = 0,\ \ \text{for } k \geq 3, \tag{91}$$

with

$$P_0(\mu) = 1\,,\ \ P_1(\mu) = \mu \ \text{ and } \ P_2(\mu) = \frac{3\mu^2 - 1}{2},\ \ \mu = \cos\theta\,. \tag{92}$$

Also, this polar equation (90) of oblate spheroid may be written in terms of Gegenbauer functions of first kind by using the following well known relation

$$\mathcal{J}_k(\mu) = \frac{P_{k-2}(\mu) - P_k(\mu)}{2k-1},\ \ k \geq 2\,,\ \ \mu = \cos\theta, \tag{93}$$

as

$$r = a\left[1-\varepsilon+2\varepsilon\,\mathcal{I}_2(\mu)\right],\ \mu = cos\ \theta, \tag{94}$$

or, if we put $d = a\ (1-\varepsilon)$, we have

$$r = d\left[1+2\varepsilon\,\mathcal{I}_2(\mu)\right]. \tag{95}$$

Now, we find the perturbation expression of Stokes drag on this axi-symmetric oblate body placed in both quadratic and stagnation like quadratic flows in axial and transverse situations with the aid of general expressions of drag [equations (20), (29), (38) and (47)].

7.1. Axial Parabolic Flow

For $\alpha \neq 1$, surface average velocity U_e in perturbed form for the surface [equation (90)] is

$$U_e = \frac{a^2}{3}(1+\alpha),$$

the value of b^2, for body surface [equation (90)], from equations (18) and (91), in perturbed form, is

$$\begin{aligned} b^2 &= a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right] \\ &= a^2\left[1-2\varepsilon+\varepsilon^2\right], \end{aligned} \tag{97}$$

since $b = a\ (1-\varepsilon)$, for oblate perturbed spheroid (see figure 3), the value of h_x, for body surface [equation (90)], from equations (19) and (91), in perturbed form, is

$$h_x = \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{4}{5}d_2\right)\right] = \frac{a}{2}\left[1-\frac{6}{5}\varepsilon\right]. \tag{98}$$

Now, for flow constant $\alpha \neq 1$, the expression of drag, F_x, from equation (20), by utilizing equations (91) and (96), comes out to be

$$\begin{aligned} F_x &= 6\pi\mu a\left[1+\varepsilon\left\{d_0-\frac{1}{5}d_2\right\}+\varepsilon^2\left\{2d_0{}^2+\frac{89}{100}d_2{}^2-\frac{21}{5}d_0d_2\right\}\right]U_e \\ &= 2\pi\mu a^3(1+\alpha)\left[1-\frac{1}{5}\varepsilon+\frac{349}{25}\varepsilon^2+\cdots\right]. \end{aligned} \tag{99}$$

For α =1, equation (99) reduces into the form

$$F_x = 4\pi\mu a^3\left[1-\frac{1}{5}\varepsilon+\frac{349}{25}\varepsilon^2+\cdots\right], \tag{100}$$

which further reduces to $4\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in axial quadratic flow [Chwang and Wu (1975), part 3], as $\varepsilon \to 0$.

If we consider the polar equation of oblate spheroid involving second order perturbation terms [see Chang and Keh, 2009] as

$$r = a\left[1\text{-}\,\varepsilon\cos^2\theta\,\text{-}\,\frac{3}{2}\varepsilon^2\cos^2\theta\sin^2\theta\right], \tag{101}$$

then, for $\alpha \neq 1$, independent application of DS-conjecture [Datta and Srivastava, 1999] on surface [equation (101)] provide the axial Stokes drag as

$$\begin{aligned} F_x &= 6\pi\mu a\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]U_e \\ &= 6\pi\mu a\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}(1+\alpha)\right] \\ &= 2\pi\mu a^3(1+\alpha)\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2+....\right], \end{aligned} \tag{102}$$

for α =1, this expression reduces to the form

$$F_x = 4\pi\mu a^3\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2+\cdots\right], \tag{103a}$$

which immediately reduces to $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial parabolic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag [equations (100) and (103a)] are in disagreement because initially the polar equations [(11) and (90)] contains only first order terms in deformation parameter 'ε' while in later equation (101), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression [equation (103a)] seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fx} by normalizing the drag [equation (103a)] by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial parabolic flow, as

$$C_{F_x} = \frac{F_x}{4\pi\mu a^3} = \left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2+\cdots\right]. \tag{103}$$

7.2. Transverse Parabolic Flow

For $\beta \neq 1$, surface average velocity U_e in perturbed form for the surface [equation (90)] is

$$\begin{aligned} U_e &= \frac{b^2}{3}\left(1+\beta-e^2\right) \\ &= \frac{a^2}{3}\left(1-e^2\right)\left(\beta+1-e^2\right) = \frac{a^2}{3}\left(1-\varepsilon\right)^2\left[\beta+\left(1-\varepsilon\right)^2\right] \\ &= \frac{a^2}{3}\left[\left(1+\beta\right)-2\left(2+\beta\right)\varepsilon+\left(6+\beta\right)\varepsilon^2\right], \end{aligned} \tag{104}$$

the value of b^2, for body surface [equation (90)], from equations (101) and (91), in perturbed form, is

$$\begin{aligned} b^2 &= a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right] \\ &= a^2\left[1-2\varepsilon+\varepsilon^2\right], \end{aligned} \tag{105}$$

since $b = a\,(1-\varepsilon)$, for oblate perturbed spheroid (see figure 3), the value of h_y, for body surface [equation (90)], from equation (102), in perturbed form, is

$$h_y = \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{11}{10}d_2\right)\right] = \frac{a}{2}\left[1+\frac{2}{5}\varepsilon\right]. \tag{106}$$

Now, for flow constant $\alpha \neq 1$, the expression of drag, F_x, from equation (29), by utilizing equations (91) and (104), comes out to be

$$\begin{aligned} F_y &= 6\pi\mu a\left[1+\varepsilon\left\{d_0+\frac{1}{10}d_2\right\}+\varepsilon^2\left\{\frac{9}{25}d_0{}^2-d_0d_2\right\}\right]U_e \\ &= 2\pi\mu a^3\left[1-\frac{2}{5}\varepsilon-\frac{41}{175}\varepsilon^2+\cdots\right]\left[\left(1+\beta\right)-2\left(2+\beta\right)\varepsilon+\left(6+\beta\right)\varepsilon^2\right] \\ &= 2\pi\mu a^3\left[\begin{array}{l}\left(1+\beta\right)+\varepsilon\left\{-2\left(2+\beta\right)-\frac{2}{5}\left(1+\beta\right)\right\} \\ \quad +\varepsilon^2\left\{\left(6+\beta\right)-\frac{41}{175}\left(1+\beta\right)+\frac{4}{5}\left(2+\beta\right)\right\}+\cdots\end{array}\right]. \end{aligned} \tag{107}$$

For β=1, equation (107) reduces into the form

$$F_y = 4\pi\mu a^3\left[1-\frac{17}{5}\varepsilon+\frac{1563}{350}\varepsilon^2+\cdots\right], \tag{108}$$

which further reduces to $4\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in transverse quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$.

If we consider the polar equation of oblate spheroid involving second order perturbation terms [see Chang and Keh (2009)] as

$$r = a\left[1 - \varepsilon \cos^2\theta - \frac{3}{2}\varepsilon^2 \cos^2\theta \sin^2\theta\right], \tag{109}$$

$$\begin{aligned} F_y &= 6\pi\mu a\left[1-\frac{2}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right]U_e \\ &= 6\pi\mu a\left[1-\frac{2}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}\left(1+\beta-e^2\right)\right] \\ &= 2\pi\mu a^3\left[1+\beta-2\varepsilon+\varepsilon^2\right]\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2+\cdots\right], \\ &= 2\pi\mu a^3\left[\left(1+\beta\right)-\frac{2\left(11+6\beta\right)}{5}\varepsilon+\frac{\left(2651+621\beta\right)}{350}\varepsilon^2+\cdots\right], \end{aligned} \tag{110}$$

then, for $\beta \neq 1$, independent application of DS conjecture [Datta and Srivastava, 1999] on surface [equation (101)] provide the axial Stokes drag as for β=1, this expression reduces to the form

$$F_y = 4\pi\mu a^3\left[1-\frac{17}{5}\varepsilon+\frac{818}{175}\varepsilon^2+\cdots\right], \tag{111a}$$

which immediately reduces to $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse parabolic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag[equations (108) and (111a)] are in disagreement because initially the polar equation [equations (11) and (90)] contains only first order terms in deformation parameter 'ε' while in later equation (109), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (111a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fy} by normalizing the drag [equation (111a)] by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse parabolic flow, as

$$C_{F_y} = \frac{F_y}{4\pi\mu a^3} = \left[1-\frac{17}{5}\varepsilon+\frac{818}{175}\varepsilon^2+\cdots\right]. \tag{111b}$$

7.3. Axial Stagnation Like Quadratic Flow

The surface average velocity U_e in perturbed form for the surface [equation (90)] is

$$U_e = \frac{b^2}{3} = \frac{a^2}{3}\left(1-e^2\right) = \frac{a^2}{3}\left(1-\varepsilon\right)^2, \tag{112}$$

the value of b^2, for body surface [equation (90)], from equations (36) and (91), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right] = a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{113}$$

as it should be, due to the fact that $b = a\,(1-\varepsilon)$, for oblate perturbed spheroid (see figure 3), the value of h_x, for body surface [equation (90)], from equations (37) and (91), in perturbed form, is

$$h_x = \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{4}{5}d_2\right)\right] = \frac{a}{2}\left[1-\frac{6}{5}\varepsilon\right]. \tag{114}$$

Now, the expression of drag, F_x , from equation (38), by utilizing equations (91) and (112), comes out to be

$$\begin{aligned} F_x &= 6\pi\mu a\left[1+\varepsilon\left\{d_0-\frac{1}{5}d_2\right\}+\varepsilon^2\left\{2d_0{}^2+\frac{89}{100}d_2{}^2-\frac{21}{5}d_0d_2\right\}\right]U_e \\ &= 2\pi\mu a^3\left[1-\frac{1}{5}\varepsilon-\frac{587}{450}\varepsilon^2+\cdots\right]\left[1-2\varepsilon+\varepsilon^2\right] \\ &= 2\pi\mu a^3\left[1-\frac{11}{5}\varepsilon+\frac{143}{450}\varepsilon^2+\cdots\right], \end{aligned} \tag{115}$$

which further reduces to $2\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in axial stagnation like quadratic flow [Chwang and Wu (1975), part 3], as $\varepsilon \to 0$.

If we consider the polar equation of oblate spheroid involving second order perturbation terms [see Chang and Keh (2009)] as

$$r = a\left[1-\varepsilon\cos^2\theta-\frac{3}{2}\varepsilon^2\cos^2\theta\sin^2\theta\right], \tag{116}$$

then, independent application of DS conjecture [Datta and Srivastava (1999)] on surface [equation (101)] provide the axial Stokes drag as

$$F_x = 6\pi\mu a\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]U_e$$

$$= 6\pi\mu a\left[1-\frac{1}{5}\varepsilon+\frac{2}{175}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}(1-\varepsilon)^2\right]$$

$$= 2\pi\mu a^3\left[1-\frac{11}{5}\varepsilon+\frac{247}{175}\varepsilon^2+\cdots\right], \tag{117a}$$

which immediately reduces to $2\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial stagnation like quadratic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag [equations (115) and (117a)] are in disagreement because initially the polar equation [equations (11) and (90)] contains only first order terms in deformation parameter 'ε' while in later equation (116), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (117a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fx} by normalizing the drag (117a) by $2\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial stagnation like quadratic flow, as

$$C_{F_x} = \frac{F_x}{2\pi\mu a^3} = \left[1-\frac{11}{5}\varepsilon+\frac{247}{175}\varepsilon^2+\cdots\right]. \tag{117b}$$

7.4. Transverse Stagnation Like Quadratic Flow

The surface average velocity U_e in perturbed form for the surface [equation (90)] is

$$U_e = \frac{a^2}{3}, \tag{118}$$

the value of b^2, for body surface [equation (90)], from equations (45) and (91), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right] = a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{119}$$

for oblate perturbed spheroid (see figure 3), the value of h_x, for body surface [equation (90)], from equations (46) and (91), in perturbed form, is

$$h_y = \frac{a}{2}\left[1+\varepsilon\left(d_0-\frac{11}{10}d_2\right)\right] = \frac{a}{2}\left[1+\frac{2}{5}\varepsilon\right]. \tag{120}$$

Now, the expression of drag, F_x, from equation (38), by utilizing equations (91) and (112), comes out to be

$$\begin{aligned} F_y &= 6\pi\mu a\left[1+\varepsilon\left\{d_0+\frac{1}{10}d_2\right\}+\varepsilon^2\left\{\frac{9}{25}d_2^{\,2}-d_0 d_2\right\}\right]U_e \\ &= 6\pi\mu a\left[1-\frac{2}{5}\varepsilon-\frac{41}{175}\varepsilon^2+\cdots\right]\left[\frac{a^2}{3}\right] \\ &= 2\pi\mu a^3\left[1-\frac{2}{5}\varepsilon-\frac{41}{175}\varepsilon^2+\cdots\right], \end{aligned} \tag{121}$$

which further reduces to $2\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in transverse stagnation like quadratic flow [Chwang and Wu (1975), part 3], as $\varepsilon \to 0$.

If we consider the polar equation of oblate spheroid involving second order perturbation terms [see Chang and Keh, 2009] as

$$r = a\left[1-\varepsilon\cos^2\theta-\frac{3}{2}\varepsilon^2\cos^2\theta\sin^2\theta\right], \tag{122}$$

then, independent application of DS conjecture [Datta and Srivastava, 1999] on surface [equation (101)] provide the axial Stokes drag as

$$\begin{aligned} F_y &= 6\pi\mu a\left[1-\frac{2}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right]U_e \\ &= 6\pi\mu a\left[1-\frac{2}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right]\left[\frac{a^2}{3}\right] \\ &= 2\pi\mu a^3\left[1-\frac{2}{5}\varepsilon-\frac{9}{350}\varepsilon^2\cdots\right], \end{aligned} \tag{123}$$

which immediately reduces to $2\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse stagnation like quadratic flow, as $\varepsilon \to 0$. It is interesting to note that the second order terms in both expressions of drag [equations (121) and (123a)] are in disagreement because initially the polar equation [equations (11) and (90)] contains only first order terms in deformation parameter 'ε' while in later equation (122), the terms up to second order of deformation parameter 'ε' are considered. In that way, the expression (123a) seems to be more precise and correct as far as numerical values of drag are concerned. We can write the drag coefficient C_{Fy} by normalizing the drag (123a) by $2\pi\mu a^3$, the drag on sphere having radius 'a', placed in transverse stagnation like quadratic flow, as

$$C_{F_y} = \frac{F_y}{2\pi\mu a^3} = \left[1 - \frac{2}{5}\varepsilon - \frac{9}{350}\varepsilon^2 + ...\right]. \qquad (7.36b)$$

8. Egg-shaped Body

We consider a body of revolution whose left half is semi-sphere with radius '*b*'

$$x = b\cos t,\ r = b\sin t, \qquad \pi \le t \le \pi/2, \tag{124a}$$

and right half is semi-spheroid (prolate) having semi-major axis length '*a*' and semi-minor axis length '*b*'

$$x = a\cos t,\ r = b\sin t, \qquad \pi/2 \le t \le 0, \tag{124b}$$

placed under unbounded longitudinal or axial flow with parabolic velocity profile

$$U_{paraboloid} = (\alpha y^2 + z^2)$$

along *x*-axis.

8.1. Axial Quadratic Flow

For $\alpha \neq 1$, the surface average velocity U_e over egg-shaped body [equations (124a,b)], in perturbed form, is

$$\begin{aligned} U_e &= \frac{b^2}{3} + \frac{b^2}{6}(1+\alpha) = \frac{b^2}{6}(3+\alpha) \\ &= \frac{a^2}{6}(1+\alpha)\left(1 - 2\varepsilon + \varepsilon^2\right), \left[by\ u\sin g \quad \frac{b^2}{a^2} = 1 - e^2 = (1-\varepsilon)^2\right], \end{aligned} \tag{125}$$

the value of b^2, for body surface [equations (124a, b)], from equations (18) and (56), in perturbed form, is

$$b^2 = a^2\left[1 + 2\varepsilon\left(d_0 - \frac{1}{2}d_2\right) + \varepsilon^2\left(d_0{}^2 + \frac{1}{4}d_2{}^2 - d_0 d_2\right)\right] = a^2\left[1 - 2\varepsilon + \varepsilon^2\right], \tag{126}$$

for prolate perturbed spheroidal right half portion (see figure 2), the value of h_x, for body surface [equations (124a, b)], from equation (13), in perturbed form, is

$$h_x = \frac{b}{4} + \frac{a}{4}\left[1 + \varepsilon\left(d_0 - \frac{4}{5}d_2\right)\right] = \frac{a}{2}\left[1 - \frac{11}{10}\varepsilon\right], \left[\text{using equation (56) and } b = a(1-\varepsilon)\right]. \tag{127}$$

Now, for flow constant $\alpha\neq 1$, the expression of drag, F_x, from equation (16), by utilizing [equations (125), (126) and (127)], comes out to be

$$F_x = \frac{3\pi\mu b^2}{h_x} U_e = \pi\ \mu\ a^3 (3+\alpha)(1-\varepsilon)^4 \left[1 - \frac{11}{10}\varepsilon\right]^{-1} \\ = \pi\mu a^3 (3+\alpha)\left[1 - \frac{29}{10}\varepsilon + \frac{281}{100}\varepsilon^2 + \cdots\right]. \tag{128}$$

For $\alpha = 1$, equation (128) reduces into the form

$$F_x = 4\pi\mu a^3 \left[1 - \frac{29}{10}\varepsilon + \frac{281}{100}\varepsilon^2 + \cdots\right], \tag{129a}$$

which further reduces to $4\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in axial quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$. This expression of drag may checked by considering the average of drag on sphere [equation (48)] and prolate spheroid [equation (64)]. We can write the drag coefficient C_{Fx} by normalizing the drag [equation (129a)] by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial quadratic flow, as

$$C_{F_x} = \frac{F_x}{4\pi\mu a^3} = \left[1 - \frac{29}{10}\varepsilon + \frac{281}{100}\varepsilon^2 + \cdots\right]. \tag{129b}$$

8.2. Transverse Quadratic Flow

For $\beta \neq 1$, the surface average velocity U_e over egg-shaped body [equations (124a, b)], in perturbed form, is

$$U_e = \frac{\frac{1}{3}(1+\beta)b^2 + \frac{1}{3}\left(1+\beta-e^2\right)b^2}{2} = \frac{b^2}{6}\left[2(1+\beta) - e^2\right] \\ = \frac{a^2}{6}(1-\varepsilon)^2\left[(1+2\beta) + (1-\varepsilon)^2\right] \\ = \frac{a^2}{6}\left[2(1+\beta) - 2\varepsilon(3+2\beta) + \varepsilon^2(7+2\beta)\right], \tag{130}$$

the value of b^2, for body surface [equations (124a, b)], from equations (29) and (56), in perturbed form, is

$$b^2 = a^2\left[1 + 2\varepsilon\left(d_0 - \frac{1}{2}d_2\right) + \varepsilon^2\left(d_0{}^2 + \frac{1}{4}d_2{}^2 - d_0 d_2\right)\right] = a^2\left[1 - 2\varepsilon + \varepsilon^2\right], \tag{131}$$

for prolate perturbed spheroidal right half portion (see figure 2), the value of h_y, for body surface [equations (124a, b)], from equation (24), in perturbed form, is

$$h_y = \frac{b}{4} + \frac{a}{4}\left[1 + \varepsilon\left(d_0 - \frac{11}{10}d_2\right)\right]$$
$$= \frac{a}{2}\left[1 - \frac{7}{10}\varepsilon\right], \qquad \left[\text{using equation (56) and } b = a(1-\varepsilon)\right]. \tag{132}$$

Now, for flow constant $\beta \neq 1$, the expression of drag, F_y, from equation (25), by utilizing [equations (130), (131) and (132), comes out to be

$$F_y = \frac{3\pi\mu b^2}{h_y}U_e = \pi\,\mu\,a^3(1-\varepsilon)^2\left[1 - \frac{7}{10}\varepsilon\right]^{-1}\left[2(1+\beta) - 2\varepsilon(3+2\beta) + \varepsilon^2(7+2\beta)\right]$$
$$= \pi\,\mu\,a^3\left[2(1+\beta) - \frac{1}{5}\varepsilon(43+33\beta) + \frac{3}{50}\varepsilon^2(123\beta - 17) + \cdots\right]. \tag{133}$$

For $\beta=1$, equation (133) reduces into the form

$$F_y = 4\pi\mu a^3\left[1 - \frac{19}{5}\varepsilon + \frac{159}{100}\varepsilon^2 + \cdots\right], \tag{134a}$$

which further reduces to $4\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in transverse quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$. This expression of drag may be checked by considering the average of drag on sphere [equation (49) and prolate spheroid (72)]. We can write the drag coefficient C_{Fy} by normalizing the drag [equation (134a)] by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial quadratic flow, as

$$C_{F_y} = \frac{F_y}{4\pi\mu a^3} = \left[1 - \frac{19}{5}\varepsilon + \frac{159}{100}\varepsilon^2 + \cdots\right]. \tag{134b}$$

8.3. Axial Stagnation Like Quadratic Flow

The surface average velocity U_e over egg-shaped body [equations (124a, b)], in perturbed form, is

$$U_e = \frac{b^2}{6} + \frac{a^2}{6}\left(1 - e^2\right) = \frac{b^2}{3}$$
$$= \frac{a^2}{6}\left(1 - 2\varepsilon + \varepsilon^2\right), \left[\text{by using } \frac{b^2}{a^2} = 1 - e^2 = (1-\varepsilon)^2\right], \tag{135}$$

the value of b^2, for body surface [equations (124a, b)], from equations (36) and (56), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right]$$
$$= a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{136}$$

for prolate perturbed spheroidal right half portion (see figure 2), the value of h_x, for body surface [equations (124a, b)], from equation (13), in perturbed form, is

$$h_x = \frac{b}{4}+\frac{a}{4}\left[1+\varepsilon\left(d_0-\frac{4}{5}d_2\right)\right]$$
$$= \frac{a}{2}\left[1-\frac{11}{10}\varepsilon\right], \qquad \left[\text{using equation (56) and } b = a\left(1-\varepsilon\right)\right]. \tag{137}$$

Now, the expression of drag, F_x, from equation (34), by utilizing [equations (135), (136) and (137)], comes out to be

$$F_x = \frac{3\pi\mu b^2}{h_x}U_e = 2\pi\ \mu\ a^3\left(1-\varepsilon\right)^4\left[1-\frac{11}{10}\varepsilon\right]^{-1}$$
$$= 2\pi\mu a^3\left[1-\frac{29}{10}\varepsilon+\frac{281}{100}\varepsilon^2+\cdots\right], \tag{138a}$$

which further reduces to $2\pi\mu a^3$, the classical drag for sphere having radius '*a*' placed in axial quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$. This expression of drag may checked by considering the average of drag on sphere equation (48) and prolate spheroid equation (64). We can write the drag coefficient C_{Fx} by normalizing the drag [equation (138a)] by $2\pi\mu a^3$, the drag on sphere having radius '*a*', placed in axial quadratic flow, as

$$C_{F_x} = \frac{F_x}{4\pi\mu a^3} = \left[1-\frac{29}{10}\varepsilon+\frac{281}{100}\varepsilon^2+\cdots\right]. \tag{138b}$$

8.4. Transverse Stagnation Like Quadratic Flow

The surface average velocity U_e over egg-shaped body [equations (124a, b)], in perturbed form, is

$$U_e = \frac{b^2}{6}\left[2-e^2\right] = \frac{a^2}{6}\left(1-\varepsilon\right)^2\left[1+\left(1-\varepsilon\right)^2\right] = \frac{a^2}{3}\left[1-3\varepsilon+\frac{7}{2}\varepsilon^2\right], \tag{139}$$

the value of b^2, for body surface [equations (124a, b)], from equations (27) and (56), in perturbed form, is

$$b^2 = a^2\left[1+2\varepsilon\left(d_0-\frac{1}{2}d_2\right)+\varepsilon^2\left(d_0{}^2+\frac{1}{4}d_2{}^2-d_0d_2\right)\right]= a^2\left[1-2\varepsilon+\varepsilon^2\right], \tag{140}$$

for prolate perturbed spheroidal right half portion (see figure 2), the value of h_y, for body surface [equations (124a, b)], from equation (24), in perturbed form, is

$$h_y = \frac{b}{4}+\frac{a}{4}\left[1+\varepsilon\left(d_0-\frac{11}{10}d_2\right)\right]$$
$$= \frac{a}{2}\left[1-\frac{7}{10}\varepsilon\right], \qquad \left[\text{using equation (56) and } b = a(1-\varepsilon)\right]. \tag{141}$$

Now, the expression of drag, F_y, from equation (43), by utilizing [equations (139), (140) and (141)], comes out to be

$$F_y = \frac{3\pi\mu b^2}{h_y}U_e = \pi\,\mu\,a^3(1-\varepsilon)^2\left[1-\frac{7}{10}\varepsilon\right]^{-1}\left[1-3\varepsilon+\frac{7}{2}\varepsilon^2\right]$$
$$= 2\pi\mu a^3\left[1-\frac{57}{10}\varepsilon+\frac{1449}{100}\varepsilon^2+\cdots\right], \tag{142a}$$

which further reduces to $2\pi\mu a^3$, the classical drag for sphere having radius 'a' placed in transverse quadratic flow [Chwang and Wu, 1975, part 3], as $\varepsilon \to 0$. This expression of drag may be checked by considering the average of drag on sphere [equation (51)] and prolate spheroid [equation (87a)]. We can write the drag coefficient C_{Fy} by normalizing the drag (8.11a) by $4\pi\mu a^3$, the drag on sphere having radius 'a', placed in axial quadratic flow, as

$$C_{F_y} = \frac{F_y}{2\pi\mu a^3} = \left[1-\frac{57}{10}\varepsilon+\frac{1449}{100}\varepsilon^2+\cdots\right]. \tag{142b}$$

9. Numerical Discussion

The numerical values of the drag coefficients for various axi-symmetric deformed bodies with respect to deformation parameter 'ε', aspect ratio 'b/a' and eccentricity 'e' are given in tables 1 to 3. The corresponding variation of the drag coefficients and their ratio is depicted via graphs in figures 4 to 9. It is interesting to note that the perturbation results of drag reported in this paper is in good agreement for small values of the deformation parameter 'ε' up to 0.5. For $\varepsilon = 0$ to $\varepsilon = 0.5$, drag values on bodies deviating from sphere reduces steadily while near ε =1, i.e., the limiting case of needle shaped body in prolate spheroid (see Table 1), drag values comes out to be 0.81143 and 0.4743 for axial and transverse parabolic flow and 0.21143 and 0.5743 for axial and transverse stagnation like parabolic flow respectively. On the other hand, for oblate spheroid, near ε =1, i.e., the case of a flat circular disk (see Table 2), drag values comes out to be 0.81143 and 2.2743 for axial and transverse stagnation like parabolic flow and 0.21143 and 0.5743 for axial and transverse stagnation like parabolic flow respectively. For these limiting values of drag, nothing is available for validation in literature. In that way, for $\varepsilon = 0$ to $\varepsilon = 0.5$, the drag values decreases from value of sphere accordingly in same manner which we are having for uniform axial and transverse Stokes flow [see Srivastava et al. (2012)].

Table 1. Numerical values of drag coefficients and their ratio with respect to deformation parameter, aspect ratio and eccentricity for prolate perturbed spheroid in parabolic and stagnation like parabolic flow (Figures 4 and 5 are based on this table)

$e=\sqrt{1-\frac{b^2}{a^2}}$	$\frac{b}{a}=1-\varepsilon$	ε	Parabolic Flow			Stagnation like parabolic flow		
			C_{Fx} eq. (67b)	C_{Fy} eq. (75b)	C_{Fx}/C_{Fy}	C_{Fx} eq. (81b)	C_{Fy} eq. (87b)	C_{Fx}/C_{Fy}
0.00000	1.0	0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.43589	0.9	0.1	0.74611	0.85074	0.87701	0.92011	0.76174	1.20790
0.60000	0.8	0.2	0.54445	0.72297	0.75308	0.84045	0.56697	0.95323
0.71414	0.7	0.3	0.39502	0.61668	0.64056	0.76102	0.41568	1.83077
0.80000	0.6	0.4	0.29782	0.53188	0.55994	0.68182	0.30788	2.21455
0.86602	0.5	0.5	0.25285	0.46857	0.53970	0.60285	0.24357	2.06451
0.91651	0.4	0.6	0.26011	0.42674	0.60953	0.52411	0.22274	2.35299
0.95393	0.3	0.7	0.31960	0.40640	0.78641	0.44560	0.24540	1.81581
0.97979	0.2	0.8	0.43131	0.40754	1.05832	0.36731	0.31154	1.17901
0.99498	0.1	0.9	0.59525	0.43017	1.38376	0.28925	0.42117	0.68679
1.00000	0.0	1.0	0.81142	0.47428	1.71084	0.21142	0.57428	0.36815

Table 2. Numerical values of drag coefficients and their ratio with respect to deformation parameter, aspect ratio and eccentricity for oblate perturbed spheroid in parabolic and stagnation like parabolic flow (Figures 6 and 7 are based on this table)

$e=\sqrt{1-\frac{b^2}{a^2}}$	$\frac{b}{a}=1-\varepsilon$	ε	Parabolic Flow			Stagnation like parabolic flow		
			C_{Fx} eq. (103b)	C_{Fy} eq. (111b)	C_{Fx}/C_{Fy}	C_{Fx} eq. (117b)	C_{Fy} eq. (123b)	C_{Fx}/C_{Fy}
0.00000	1.0	0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.43589	0.9	0.1	0.98011	0.70674	1.38680	0.79411	0.95974	0.82742
0.60000	0.8	0.2	0.96045	0.50697	1.89450	0.61645	0.91897	0.67081
0.71414	0.7	0.3	0.94102	0.40068	2.34854	0.46702	0.87768	0.53211
0.80000	0.6	0.4	0.92182	0.38788	2.37654	0.34582	0.83588	0.41372
0.86602	0.5	0.5	0.90285	0.46857	1.92683	0.25285	0.79357	0.31863
0.91651	0.4	0.6	0.88411	0.64274	1.37553	0.18811	0.75074	0.25057
0.95393	0.3	0.7	0.86560	0.91040	0.95079	0.15160	0.70740	0.21430
0.97979	0.2	0.8	0.84731	1.27154	0.66636	0.14331	0.66354	0.21598
0.99498	0.1	0.9	0.82925	1.72617	0.48040	0.16325	0.61917	0.26367
1.00000	0.0	1.0	0.81142	2.27428	0.35678	0.21143	0.57428	0.36816

Table 3. Numerical values of drag coefficients and their ratio with respect to deformation parameter, aspect ratio and eccentricity for egg-shaped body in parabolic and stagnation like parabolic flow (Figures 8 and 9 are based on this table)

$e=\sqrt{1-\frac{b^2}{a^2}}$	$\frac{b}{a}=1-\varepsilon$	ε	Parabolic Flow			Stagnation like parabolic flow		
			C_{Fx} eq. (123b)	C_{Fy} eq. (134b)	C_{Fx}/C_{Fy}	C_{Fx} eq. (138b)	C_{Fy} eq. (142b)	C_{Fx}/C_{Fy}
0.00000	1.0	0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.43589	0.9	0.1	0.73890	0.82590	0.89466	0.73810	0.57490	1.28387
0.60000	0.8	0.2	0.53560	0.68360	0.78349	0.53240	0.43960	1.21110
0.71414	0.7	0.3	0.39010	0.57310	0.68068	0.38290	0.59410	0.64450
0.80000	0.6	0.4	0.30240	0.49440	0.61165	0.28960	1.03840	0.27889
0.86602	0.5	0.5	0.27250	0.44750	0.60893	0.25250	1.17725	0.21448
0.91651	0.4	0.6	0.30010	0.43240	0.69472	0.27160	2.79640	0.09712
0.95393	0.3	0.7	0.38610	0.44910	0.85971	0.34690	4.11010	0.08440
0.97979	0.2	0.8	0.52960	0.49760	1.06430	0.47840	5.71360	0.08373
0.99498	0.1	0.9	0.73090	0.57790	1.26475	0.66610	7.60690	0.08756
1.00000	0.0	1.0	0.99000	0.69000	1.43478	0.91000	9.79000	0.09295

10. Conclusion

By using DS conjecture (Datta and Srivastava, 1999) and the concept of surface average velocity proposed by (Chwang and Wu, part 2 and 3, 1975), a novel perturbation method for a steady quadratic Stokes flow past deformed axi-symmetric bodies is presented in this paper. The method (Datta and Srivastava, 1999) has been advanced by replacing uniform velocity U_x and U_y by surface average velocity over the considered surface. The class of spheroidal bodies are considered together with an egg-shaped body (combination of semi-spherical and semi-spheroidal bodies for which the condition of continuous turning tangent satisfies at the junction point) for application of proposed method. It has been observed that trends ofthe drag in every flow situations are in agreement with that of uniform Stokes flow past deformed sphere (see Srivastava et al., 2012) for ε=0 to ε=0.5. While for large departures of ε towards 1.0 things look uncertain as no values are available in the literature for quadratic Stokes flow. Author is trying to explore further avenues for some more complex flows like extensional (hyperboloidal), shear… and non-linear quadratic Stokes flow.

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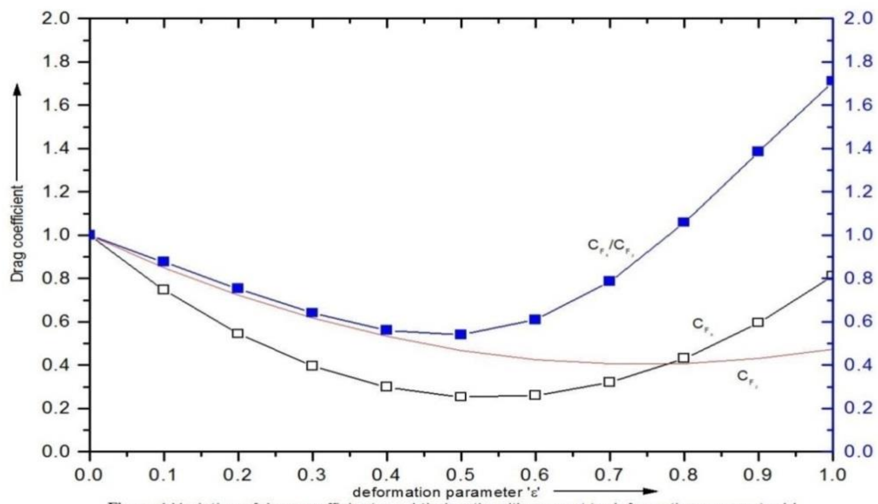

Figure 4 Variation of drag coefficients and their ratio with respect to deformation parameter 'ε' for prolate perturbed spheroid in parabolic flow

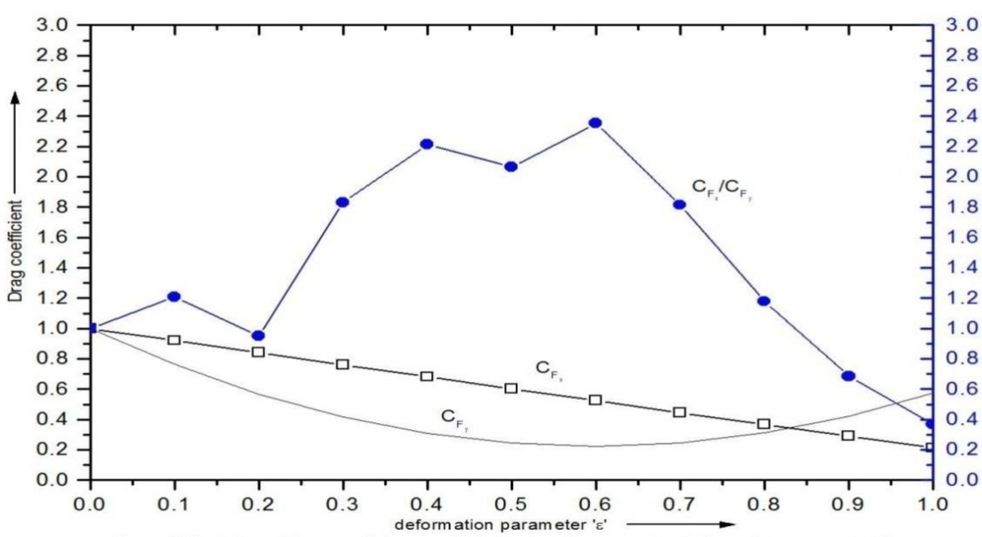

Figure 5 Variation of drag coefficients and their ratio with respect to deformation parameter 'ε' for prolate perturbed spheroid in stagnation like parabolic flow

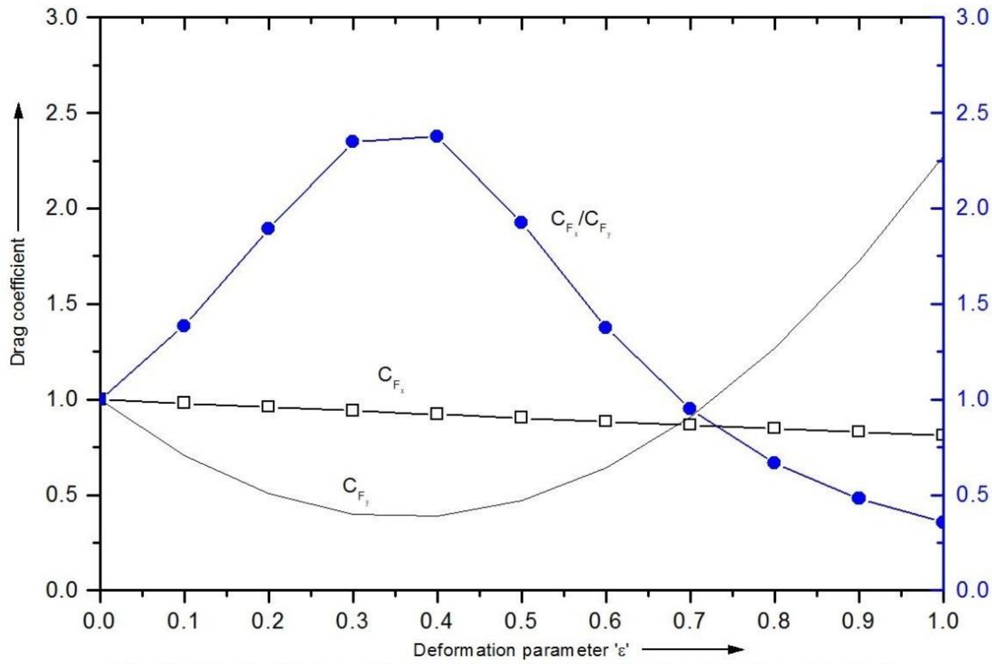

Figure 6 Variation of drag coefficients and their ratio with respect to deformation parameter 'ε' for oblate spheroid in parabolic flow

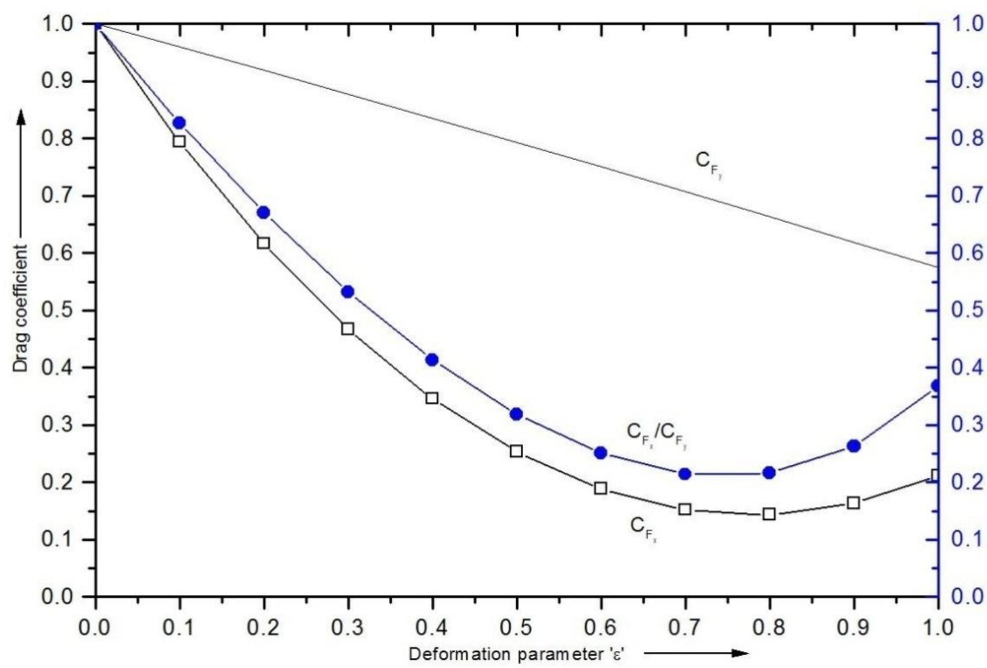

Figure 7 Variation of drag coefficients and their ratio with respect to deformation parameter 'ε' for oblate spheroid in stagnation like parabolic flow

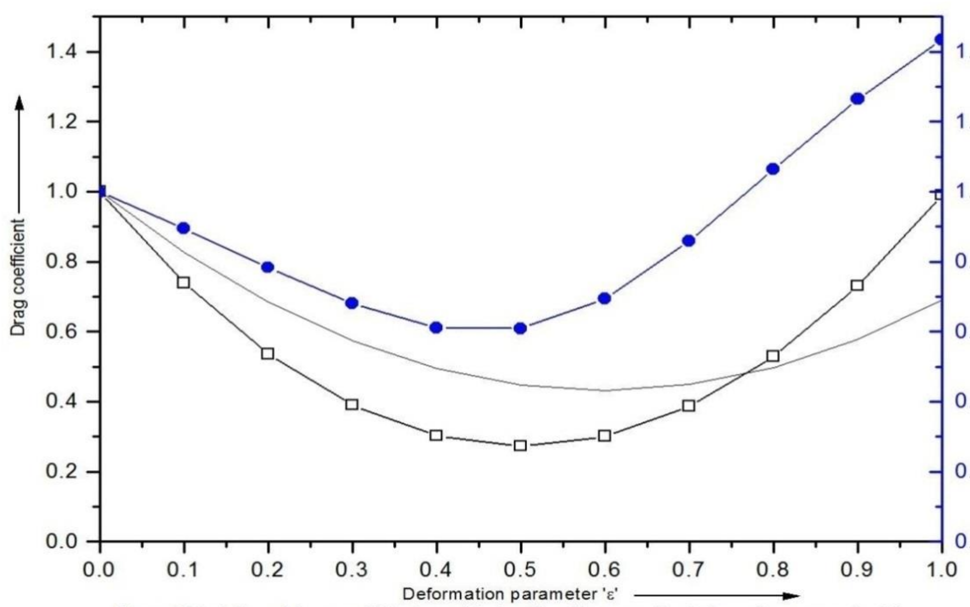

Figure 8 Variation of drag coefficients and their ratio with respect to deformation parameter 'ε' for egg-shaped body in parabolic flow

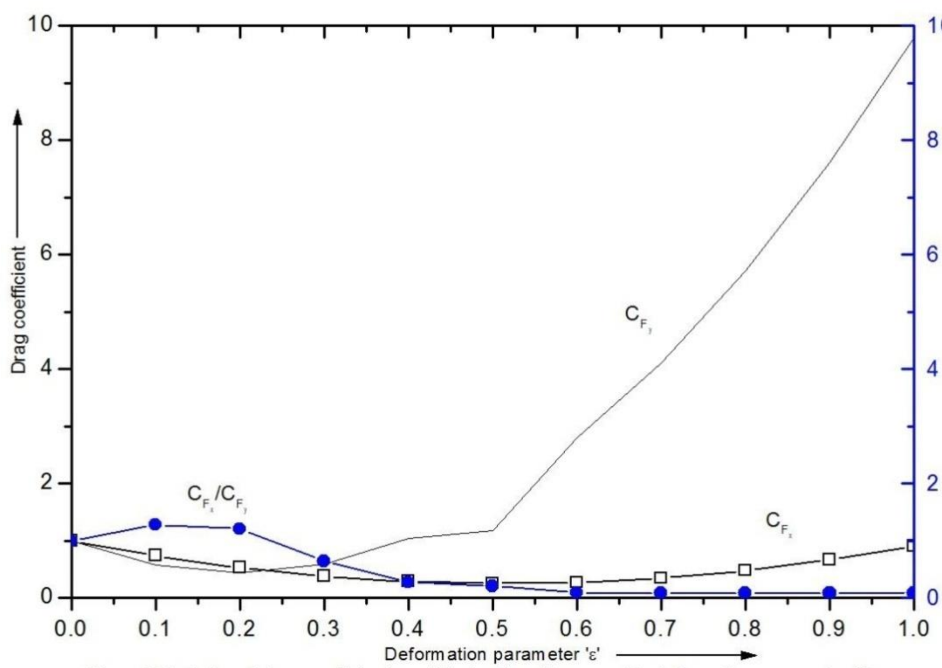

Figure 9 Variation of drag coefficients and their ratio with respect to deformation parameter 'ε' for egg-shaped body in stagnation like parabolic flow